Integrating Sustainability into Algebra Courses

A Manual for Instructors

Rikki Wagstrom
Metropolitan State University

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An initiative of the National Center for Science and Civic Engagement, Engaging Mathematics applies the well-established SENCER method to college level mathematics courses, with the goal of using civic issues to make math more relevant to students.

Engaging Mathematics will: (1) develop and deliver enhanced and new mathematics courses and course modules that engage students through meaningful civic applications, (2) draw upon state-of-the-art curriculum in mathematics, already developed through federal and other support programs, to complement and broaden the impact of the SENCER approach to course design, (3) create a wider community of mathematics scholars within SENCER capable of implementing and sustaining curricular reforms, (4) broaden project impacts beyond SENCER by offering national dissemination through workshops, online webinars, publications, presentations at local, regional, and national venues, and catalytic site visits, and (5) develop assessment tools to monitor students' perceptions of the usefulness of mathematics, interest and confidence in doing mathematics, growth in knowledge content, and ability to apply mathematics to better understand complex civic issues.

Updates and resources developed throughout the initiative will be available online at www.engagingmathematics.net. Follow the initiative on Twitter: @MathEngaging.

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Chapter 1. Preliminary Information

Welcome!

This manual discusses four classroom-tested, sustainability-themed modules that may be used collectively or individually in mathematics courses ranging from intermediate algebra through pre-calculus and math for liberal arts.

Organization of the Manual

This manual is organized into five chapters. Chapter 1 provides detailed information about the *Mathematics of Sustainability* course offered at Metropolitan State University, and illustrates how the modules have been used collectively to integrate sustainability topics throughout an entire mathematics course.

The modules can also be used individually as projects in intermediate algebra, college algebra, pre-calculus, or math for liberal arts courses. Chapters 2-5 discuss the individual modules and include the curriculum. Each chapter begins with an overview of the module and instructor resources. The actual curriculum follows.

The Modules

The four modules discussed in this manual explore the following topics:

- Milkweed and monarch butterflies (two-part module)
- Wind energy (two-part module)
- Greenhouse gas emissions associated with automotive fuels
- Carbon footprints and population growth

All of the modules require students to apply and interpret mathematics in real-world contexts, and use numeracy skills and quantitative reasoning such as arithmetic with large numbers (using scientific notation), rounding, understanding units and performing unit conversions, calculating percentage change, and reading graphs. Some of the modules target particular algebraic skills, while others emphasize general quantitative thinking. The table on the next page provides a brief description of the mathematical skills targeted in each of the modules. The individual chapters in this manual discuss the modules in greater depth.

The modules each conclude with a set of homework exercises. In most cases, one of the exercises is a research question that prompts students to learn more about a module topic.

The modules contain a significant amount of scaffolding enabling them to be used by less-experienced students. Instructors teaching more advanced mathematics courses are free to remove the scaffolding and pose more open-ended questions.

Module Titles	Targeted Skills
Common Milkweed: What about it?	Quantitative thinking
The Monarch and the Milkweed	Deriving and applying exponential functions, extrapolation
How much energy can a wind turbine produce?	Deriving and applying multivariable functions
Wind energy: What does it cost?	Quantitative thinking
Does my car emit greenhouse gases?	Deriving and applying multivariable functions
Exploring Carbon Dioxide Emissions in the United States	Deriving and applying linear and quadratic functions, extrapolation

Things to Consider Before Using the Modules

Some of the modules included in this manual require a significant amount of reading. The reading is frequently of a technical nature and often includes new vocabulary. Several of the modules are lengthy and layered, requiring students to recall and apply concepts and results from earlier sections of the modules. Finally, the modules require quantitative reasoning, modeling, and/or mathematical skills that are new for many students. Collectively, these aspects of the modules can occasionally make them seem overwhelming and confusing to a student.

Prior to using these modules instructors will want to consider how best to address these challenges in their own classes. Below are some suggested approaches:

- For modules that have introductory reading, consider assigning the reading and the first few module exercises as a pre-class assignment. This enables students to read and consider the ideas at their own pace. This can be particularly helpful for students that are non-native English speakers. Students might then come to class prepared to discuss the first exercises with peers.

- Consider having a brief conversation with students about reading skills. In this age of social media, students are becoming increasingly accustomed to short, simple blocks of text that can be quickly and easily read and processed. However, to make sense of the topics presented in these modules, students will need to slow down and read carefully.

- Consider dividing a module into smaller pieces with formative assessments associated with each piece. Given the layered nature of the modules, students benefit from frequent feedback on their work enabling them to correct mistakes or misconceptions but also to reflect on and discuss what they learned before moving on. Formative assessments might take the form of short oral summaries to the class or write-ups of individual module exercises.

Copyright and Use of this Curriculum

Instructors may not post solution keys online for any of the curriculum included in this manual. Instructors are also discouraged from distributing hard copies of solution keys to their students.

Acknowledgements

One of the modules included in this manual, *Does my car emit greenhouse gases?*, is co-authored with Jodin Morey. At the time this module was written, Jodin was finishing a mathematics secondary education major at Metropolitan State University. He did the background research for this module and wrote the initial drafts. I wrote the introductory section of the module and later revisions.

The following individuals reviewed and/or piloted the curriculum and provided invaluable feedback which informed the final version of this manual:

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Many thanks to all of you for your time and support of this project.

Chapter 2. Mathematics of Sustainability

The modules discussed in this manual were developed for use in a non-traditional course offered at Metropolitan State University entitled *Mathematics of Sustainability*. As a prerequisite for all general education mathematics courses including college algebra, statistics, and math for liberal arts, *Mathematics of Sustainability* offers students an alternative to a standard intermediate algebra course. The integration of significant sustainability-themed content into the course allows students enrolling in the course to earn general education credits.

Students enrolling in *Mathematics of Sustainability* need to catch on to mathematical content fairly quickly and independently to devote greater time in class to working on the sustainability modules. Consequently, these students are required to have higher scores on their mathematics placement test, specifically a score of 95 or higher on either the Accuplacer Arithmetic or Elementary Algebra assessments, than are required for our standard intermediate algebra course (which requires a minimum score of 39 on the Accuplacer Arithmetic assessment).

The *Mathematics of Sustainability* course uses a flipped-classroom pedagogy. Students learn the mathematical content outside of class with the assistance of a workbook and accompanying screencasts developed specifically for the course. The workbook chapters are shown in the table below. The first three chapters of the workbook cover selected numeracy and quantitative reasoning skills necessary for studying real-world problems. These topics are referenced throughout the course.

Descriptions of Workbook Chapters	
Chapter 1: Working with Large and Small Numbers	Exponent properties, scientific notation, rounding
Chapter 2: Quantifying Change	Total change and percent change, growth and decay factors
Chapter 3: Working with Units	SI prefixes, unit conversion, back-of-the-envelope estimation
Chapter 4: Introduction to Functions	Order of operations, graphs and equations of functions, intercepts
Chapter 5: Rate of Change	Slope, practical interpretations
Chapter 6: Expressions, Equations, and Inequalities	Properties of real numbers, simplifying, expanding, and factoring expressions, solving linear equations and inequalities
Chapter 7: Recursion	Introduction to linear and exponential processes, setting up initial value problems in applied contexts
Chapter 8: Linear Functions	Modeling linear processes, deriving equations of lines, finding points of intersection
Chapter 9: Exponential Functions	Modeling exponential processes, exponent properties and roots
Chapter 10: Logarithmic Functions	Finding inverses of linear and exponential functions, solving equations involving exponential functions
Chapter 11: Quadratic Functions	Modeling quadratic phenomena, finding vertices, finding horizontal and vertical intercepts, and graphing functions

At Metropolitan State University, classes generally meet once a week. Class meetings for four-credit-hour courses like *Mathematics of Sustainability* are three hours and twenty minutes. The first half of a typical class involves discussion of the mathematical topic students have worked on over the previous week, followed by a quiz covering this topic. The second half of class is frequently devoted to the sustainability-themed modules. Students work through the in-class portion of the modules in small groups, and then complete the accompanying homework exercises outside of class. The table below shows a schedule for the fifteen-week course using the sustainability-themed modules included in this manual.

Mathematics of Sustainability Schedule		
Week	**In-class Work**	**Workbook Assignment**
1	Introduction to the course Begin *Common Milkweed: What about it?*	Chapters 1 and 2
2	Discussion and Quiz over Chapters 1 and 2 Finish *Common Milkweed: What about it?*	Chapter 3
3	Discussion and Quiz over Chapter 3 Begin *How much energy does a wind turbine produce?*	Chapter 4
4	Discussion and Quiz over Chapter 4 Continue *How much energy does a wind turbine produce?*	
5	Exam 1 Finish *How much energy does a wind turbine produce?*	Chapter 5
6	Discussion and Quiz over Chapter 5 Begin *Does my car emit greenhouse gases?*	Chapter 6
7	Discussion and Quiz over Chapter 6 Finish *Does my car emit greenhouse gases?*	Chapter 8
8	Discussion and Quiz over Chapter 8 Review for Exam 2	
9	Exam 2 Begin *Wind Energy: What does it cost?*	
10	Finish *Wind Energy: What does it cost?* Begin *Exploring Carbon Dioxide Emissions in the United States*	Chapter 11
11	Discussion and Quiz over Chapter 11 Finish *Exploring Carbon Dioxide Emissions in the United States*	Chapter 9
12	Discussion and Quiz over Chapter 9 Complete *The Monarch and the Milkweed*	Chapter 9
13	Discussion and Quiz over Chapter 9 Introduction to inverse functions and logarithms	Chapter 10
14	Discussion and Quiz over Chapter 10 Review for Exam 3	
15	Exam 3	

Chapter 3. Milkweed and Monarch Butterflies

This is a two-part module. Although Part A is intended as preparation for Part B, each part is self-contained and can be used separately. Because there is no algebraic content in Part A, *Common Milkweed: What about it?,* it can be used in remedial mathematics, quantitative reasoning, or math for liberal arts courses. Part B, *The Monarch and the Milkweed*, focuses on the topic of exponential growth and decay and would therefore be appropriate for college algebra, pre-calculus, or math for liberal arts courses.

Instructor Notes for Part A: *Common Milkweed: What about it?*

Synopsis: This activity explores changes in milkweed density in Iowa corn and soybean fields between 1999 and 2009 due to increased use of chemical herbicides applied in these fields. Iowa is singled out because it is the only state for which such data exists. However, similar losses are believed to be occurring in other corn and soybean producing states. Students are presented with a square representing a one-hectare corn field. On this square, they draw rectangles representing the average square meter areas occupied by milkweed in 1999 and 2009. There are two key observations: (1) the percentage of a typical hectare corn field occupied by milkweed in 1999 is very small, and (2) the area occupied by milkweed has decreased significantly. The first observation begs the question of why herbicides are necessary and prompts students to consider the perspective of a farmer. The second observation captures the perspective of a female monarch butterfly struggling to locate decreasing numbers of milkweed plants on which to lay her eggs. Students then consider the idea that sustainable solutions to problems necessarily require people to consider multiple points of view.

Skills/knowledge Required: Areas of rectangles, estimation, percents, percentage change, reading graphs

Vocabulary Used: milkweed, density, hectare, herbicide, herbicide-tolerant plant

Structuring In-Class and Out-of-Class Work: Students in the *Mathematics of Sustainability* course at Metropolitan State University required approximately 2 hours, working in groups of 3-4 students, to complete the activity in class. Students spent considerable time on questions 1 and 2. Some instructors might prefer to assign these first two questions as a pre-class assignment, to give students an opportunity to think through these questions independently before conferring with their peer group during class. There are three homework exercises at the conclusion of this activity. Any subset of these can constitute a post-class homework assignment.

Note: For more advanced mathematics courses, this activity could be done as a take-home assignment in preparation for doing Part B: *The Monarch and the Milkweed* in class.

Additional Notes:

- Exercise 1 presents two challenges. The first challenge is choosing side lengths for a rectangle whose area is approximately 23 m^2. The second challenge is accurately embedding the rectangle within the one-hectare field. Students frequently assume that the tick marks along the perimeter of the field represent 1-meter distances. Even when students recognize that the distance between tick marks represents 10 meters, they sometimes find it challenging to accurately mark shorter distances, such as 2 meters. After working exercise 1, instructors might do a formative assessment to correct any initial misconceptions.

- Exercise 3 asks students to explain what the milkweed density values mean. This exercise is frequently confusing for students, but it is essential that they be able to answer this question thoroughly and correctly in order to understand the story of milkweed in the United States. Instructors might consider alternative ways of explaining this exercise when more guidance is needed. This is also a place where instructors might include a formative assessment.

- Instructors wanting additional information on genetically engineered crops and herbicide use are directed to the USDA site listed below.

Web Resources:

Site	url
Xerces	http://www.xerces.org/milkweed/
Monarch Watch	http://www.monarchwatch.org/milkweed/
Monarch Joint Venture	http://monarchjointventure.org/get-involved/
United States Department of Agriculture Economic Research Service	http://www.ers.usda.gov/amber-waves/2014-march/adoption-of-genetically-engineered-crops-by-us-farmers-has-increased-steadily-for-over-15-years

Instructor Notes for Part B: *The Monarch and the Milkweed*

Synopsis: This activity begins by exploring the amazing reproductive potential of monarchs. Assuming a female monarch and all of her female offspring survive to reproduce, a population of monarchs will grow exponentially at a very large rate. Students determine an exponential function describing this population growth over time and use their function to observe enormous population growth over a year. This growth is contrasted with the decreasing population trend observed over the past twenty years at the Monarch Biosphere Reserve in Mexico, suggesting that pre-reproductive mortality among monarchs is quite high. The activity then explores one factor believed to be a significant cause of monarch population decline, namely decreasing milkweed populations in the United States. Using data from studies in Iowa, students examine decreases in milkweed densities in crop fields, pastures, roadsides, and land set aside in conservation programs. Among most land types, milkweed densities have decreased roughly exponentially between 1999 and 2009. Students derive exponential functions estimating milkweed densities among the different land types, and then use their functions to determine projected densities in 2014. Using data on Iowa land area usage and monarch egg density (eggs/stem), and making an assumption about the number of milkweed stems per square meter, students arrive at estimates for peak egg laying in Iowa in 1999 and 2014. They then calculate the percent decrease and discuss the larger significance of this decrease.

Skills/knowledge Required: Areas of rectangles, estimation, percentage change, deriving and evaluating exponential functions

Vocabulary Used: density, hectare, growth and decay factors

Structuring In-Class and Out-of-Class Work: As this activity is both lengthy and challenging, instructors will want to consider carefully how best to integrate it into their course. Students in the *Mathematics of Sustainability* course at Metropolitan State University worked exercises 1-4 at the start of the activity as a pre-class assignment. It then took approximately 2 hours for the students, working in groups of 3-4, to discuss these exercises and complete the remainder of the activity in class.

The activity is divided into three main sections. Exercises 1-4 comprise the first section; exercises 5 and 6 comprise the second section; and exercises 7-9 comprise the final section. Instructors might consider doing formative assessments after each section. One possible approach is to assign the exercises from a single section as homework and use class time for discussion, feedback, and evaluation before moving on to the next section. In this way, the activity can occupy smaller blocks of time over 3-4 50-minute class meetings. There are four homework exercises at the conclusion of this activity. Any subset of these can constitute a post-activity homework assignment.

Additional Notes:

- This activity assumes that students are familiar with exponential functions of the form $y = b(R)^t$. The parameter R is referred to in the activity as an annual ``growth/decay factor".
- Students commonly ask why the activity focuses only on Iowa milkweed populations. The reason is that Iowa is the only state for which longitudinal data on milkweed density exist across different land types. The time period 1999-2009 is a significant period for observing changes in milkweed abundance in crop fields as herbicide-tolerant crops were introduced to commercial markets beginning in the late 1990's.
- Students should be encouraged to slow down and re-read exercises as they make their way through this activity.

Web Resources:

Site	url
Monarch Larva Monitoring Project	http://www.mlmp.org/
Monarch Watch	http://www.monarchwatch.org/
Monarch Joint Venture	http://monarchjointventure.org/
Journey North	https://www.learner.org/jnorth/monarch/
Monarch Butterfly Fund	http://www.monarchbutterflyfund.org/
Flight of the Butterflies	http://www.flightofthebutterflies.com/home/

Module 1A: Common Milkweed: What about it?

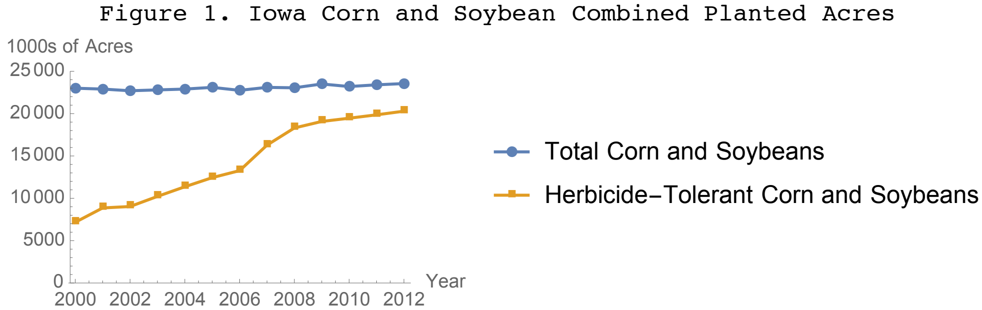
(Gorman, 2014)

The common milkweed is one of a variety of plants that grows in roadside ditches, crop fields, and pastures around the United States. You've probably seen this plant at least once in your life, but it may have escaped your notice unless you happened to see a milkweed up close either when its pink flowers stand out or when its large seed pods form and it sheds seeds.

(Caldwell, 2014)

In the late 1990s, herbicide-resistant corn and soybeans were introduced to crop farmers around the country. These genetically modified plants had the ability to withstand the application of a particular herbicide designed to kill off weeds (in other words, undesirable plants) in crop fields. So, farmers who plant the herbicide-tolerant corn and soybeans in their fields could kill off weeds (such as milkweed) without harming their crops. The graph below illustrates how the application of herbicide-tolerant corn and soybeans in Iowa has grown over time.

Figure 1. Iowa Corn and Soybean Combined Planted Acres



As a result, the presence of milkweed in corn and soybean fields has declined. In 1999 the average milkweed density in Iowa corn and soybean fields was reported to be about 23.00 m^2/hectare. In 2009, the density was reported to be about 5.00 m^2/hectare. In this activity you will explore the implications of the density decrease.

What do we mean by "milkweed density"?

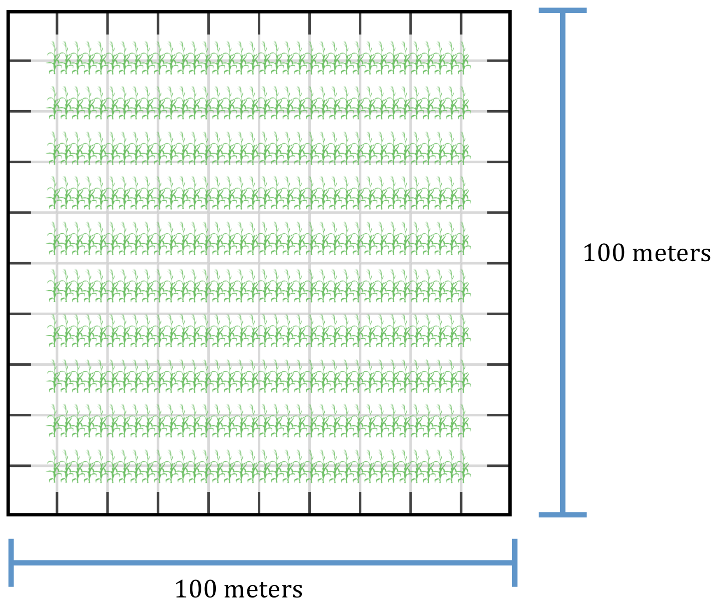
What do the quantities 23.00 m^2/hectare and 5.00 m^2/hectare mean? First of all, let's talk about hectares. A hectare is a unit of land area equal to 10,000 m^2 (ten thousand square meters). Imagine that the square shown in Figure 1 on the next page represents a one-hectare cornfield. Each side of the square represents 100 m (meters) because

$$100 \text{ m} \times 100 \text{ m} = 10{,}000 \text{ m}^2 = 1 \text{ hectare} = \mathit{Area\ of\ cornfield}.$$

Since one meter is about 3.3 feet, each side of our cornfield is actually about 330 feet. So, the square drawn on the next page represents a fairly large land area in real life.

1. Now, inside the square field, draw a rectangle whose area is approximately 23.00 m^2. Remember that the area of a rectangle is equal to the product of the rectangle's length and width. So, you just need to choose a length and a width (in meters) so that the product will be roughly 23.00 m^2.

Figure 2. A one-hectare cornfield in Iowa.



2. Now, inside the rectangle that you drew in exercise 1, draw another rectangle whose area is approximately 5.00 m^2.

3. So, when we talk about average milkweed densities of 23.00 m^2/hectare and 5.00 m^2/hectare in Iowa corn and soybean fields, what do we mean? Write 1-2 sentences describing what these two densities mean. Don't forget that these densities are specific to the years 1999 and 2009. Hint: Recall that our original square represents a one-hectare cornfield. Consider what your two smaller rectangles must represent.

4. In 1999, what percentage of our one-hectare cornfield has milkweed growing on it? What is the percentage in 2009? Remember that one hectare is equal to 10,000 m^2. Did milkweeds occupy much space in these fields in either of these years?

5. (For Discussion) If milkweeds occupy such a small percentage of land area within cornfields, why do farmers apply herbicides at all? Hint: What else might be growing in cornfields?

6. Let's look at this situation from another perspective. By what percentage did the average milkweed density in Iowa decrease over the 1999-2009 period? To calculate the percentage decrease, you subtract the densities and then divide by the original (1999) density. Move the decimal point two places to the right to express your answer as a percent. Write a one-sentence interpretation of your answer.

7. What does your answer in exercise 6 tell you about the relative sizes of the rectangles you drew in exercises 1 and 2?

8. (For Discussion) Monarch butterflies migrate to the U.S. and Canada from Mexico each spring in search of milkweed (see photo at right), the exclusive food source for monarch larvae. Locating sufficient amounts of milkweed is essential to the reproductive success of monarchs. Do you think the decrease in milkweed density in corn and soybean fields would be significant from the perspective of monarchs? Explain.

(Faria!, 2011)

9. (For Discussion) Resolving environmental issues is almost never a simple matter. There are multiple perspectives that need to be understood and respected if "sustainable" solutions are to be found. The notion of a sustainable solution can only be understood in terms of how we define "sustainable" or more generally "sustainability". Different organizations (sometimes representing different perspectives) have different definitions for this term. For example,

   - *the perpetual maintenance of diverse and productive environments upon which all life depends* (Renewable Natural Resources Foundation)
   - *meeting the needs of the present without compromising the ability of future generations to meet their own needs* (World Commission on Environment and Development)

   Would you say that these definitions are inclusive of multiple, diverse perspectives?

10. (For Discussion) A central idea in this activity is the importance of considering diverse perspectives as we make decisions that have implications for the environment. How does this activity address diverse perspectives on milkweed plants?

What if this trend in milkweed loss continues?

11. Suppose milkweed densities continue to decrease in future years. Over the years 1999-2009, there was a 78% decrease in Iowa milkweed density in corn and soybean fields. Let's assume for the moment that we continue to see a 78% decrease on 10-year periods in the future.

    a. Determine the 10-year decay factor for milkweed density. Recall that the decay factor represents the percentage of milkweed density remaining after ten years.

    b. Use the 10-year decay factor to complete the following table.

Year	2009	2019	2029
Milkweed Density (m^2/hectare)	5.0		

    c. Compare the projected milkweed density in 2029 from part (b) with the 1999 density of 23.0 m^2/hectare. By what percentage will the milkweed density in Iowa corn and soybeans have fallen over these 30 years?

12. (Summary) What are some of the key observations or insights that this exploration highlights for you?

Homework Exercises

1. Using the graph in Figure 1 on the first page of this activity, determine the percentage increase in the area planted with herbicide-tolerant corn and soybeans in Iowa between 1999 and 2009.

2. In this activity, you learned how to interpret milkweed densities. In this exercise, you will reflect on the idea of density a little more. Suppose that you were told that the density of a pollutant in a lake is 0.03 milligrams per liter. Circle all of the following interpretations that you believe are correct.

    a. An average liter of the pollutant has a mass of 0.03 milligrams.
    b. An average liter of lake water will contain 0.03 milligrams of pollutant.
    c. 0.03 milligrams of lake water will contain an average of 1 liter of pollutant.

3. **Do some research**. Are there any regional or national initiatives currently underway to protect or restore milkweed populations in the U.S.? Find two different examples of milkweed initiatives. For each initiative, write a paragraph including all of the following information (if possible): description of the initiative, where the initiative is taking place, how the initiative got started, and how successful the initiative has been. You must cite your sources in order to receive credit!

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Photos Used in this Activity

"Common Milkweed (Minnesota)" by Wendy Caldwell. Retrieved from https://flic.kr/p/gaoWyM. CC BY-NC-SA 2.0, no changes were made. License: https://creativecommons.org/licenses/by-nc-sa/2.0/legalcode

"Common milkweed and friends, Spring Green Preserve" by Peter Gorman. Retrieved from https://flic.kr/p/52dFK1. CC BY-NC-SA 2.0, no changes were made. License: https://creativecommons.org/licenses/by-nc-sa/2.0/legalcode

"Future Monarch munching on milkweed" by Faria! Retrieved from https://flic.kr/p/9bQ274. CC BY-NC-SA 2.0, no changes were made. License: https://creativecommons.org/licenses/by-nc-sa/2.0/legalcode

Module 1B: The Monarch and the Milkweed

(Charron-Plante, 2007)

In a previous activity, we explored declining milkweed densities in upper mid-western crop fields (Iowa in particular) that are due in large part to the increasing application of chemical herbicides. In addition to crop fields, milkweed can be found in pastures, roadside ditches, and land enrolled in conservation programs. In this activity, we will explore the prevalence of milkweed in both agricultural and non-agricultural regions within Iowa.

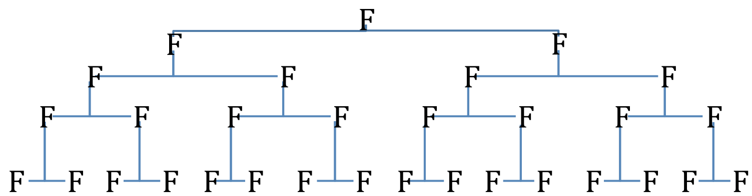
Each spring, monarch butterflies wintering in oyamel fir trees in a small region of central Mexico begin a northern migration through the eastern half of the United States, to the northern Midwest and northeastern states, and Canada, (up to 3000 miles!) to reproduce. Each fall, to escape the cold northern climates, the monarchs return to the same oyamel fir trees in Mexico. This migration requires a lot of energy. So, monarchs seek out nectar-bearing flowers (their primary source of calories) along the way to fuel their trip. Why do monarchs undertake such an immense journey to reproduce? Female monarchs lay their eggs on milkweed plants, the primary food source for monarch larvae. Since milkweed does not grow near their wintering sites, they migrate to the United States in search of this plant. Because reproductive success is key to the survival of the species, the fate of the monarch population in the future depends heavily on the fate of the milkweed population in the United States. In this activity, we explore this relationship. First, however, we'll learn a little more about monarch migration and reproduction.

Did you know that monarchs have an amazing reproductive capacity?

(Faria!, 2011)

Female monarchs lay an average of 700 eggs during their lifetime. After laying these eggs, they die; leaving a new generation to populate the species. Let's consider a female monarch that overwinters in Mexico. In the spring, she flies to the southern United States, breeds, deposits her eggs on milkweed plants, and then dies. A few weeks later, the monarch larvae mature into adult monarchs. These butterflies continue the migration up to the northern states and Canada in early-mid summer in search of milkweed. The females breed, deposit their eggs, and then die. Over the next few weeks, a new generation of adult monarchs will populate these regions of North America. Depending on the abundance of milkweed, there will be one or two additional generations produced over the summer and early fall. These adults in the final generation have the extraordinary task of making the migration south all the way back to Mexico, surviving the winter there, and then flying back to the southern regions of the U.S. again in the spring to breed and lay eggs. Whereas the monarchs from the summer generations live only 3-5 weeks, the migrating monarchs in this later generation need to live 8-9 months. So, over the course of a year, there will be 3-4 generations of monarchs, most migrating to a new geographic region in search of food and favorable habitats. Because the monarchs die shortly after reproducing, those that return to Mexico are actually 3rd or 4th generation descendants of the monarchs that overwintered in Mexico the year before.

1. Let's begin with a simple example to illustrate the relationship between the birth rate of a female (animal or plant) and the growth of a population over time. Consider a species where each female produces two female offspring during her lifetime. The tree diagram below shows four generations of female descendants originating from a single female:



(a) How many females are in each generation? Complete the following table.

Generation	0	1	2	3	4	5
Population (Number of females)	1					

(b) Write down the equation of a function that will calculate the number of female offspring P in the n^{th} generation.

(c) How would your equation need to change if each female produces three female offspring during her lifetime?

(d) How would your equation need to change if each female produces ten female offspring during her lifetime?

(e) Suppose each female has R female offspring during her lifetime. Write down the equation of a function that will calculate the number of female offspring P in the n^{th} generation.

(f) So far, we've always started from a single female in generation 0. Suppose now that you begin with 20 females. How would your equations in (b-d) need to change? Write down the new equations.

(g) Suppose that you start with P_0 females in generation 0 and all females have R female offspring during their lifetimes. Write down the equation of a function that will calculate the number of female offspring P in the n^{th} generation.

(h) What type of function describes the growth of the female population at each generation? (For example, linear, quadratic, exponential, logarithmic…)

2. Let's consider a single female adult monarch. Assume she and all of her female descendants each lay 700 eggs. Let's assume that half of the eggs are female and that all of the eggs survive to become adult monarchs. In short, each adult female produces 350 adult female offspring. Determine the equation of a function that will calculate the number of adult female monarchs in the n^{th} generation.

3. This exercise illustrates the extraordinary reproductive capacity of monarchs. Suppose that the single female monarch (generation 0) from exercise 2 overwintered in Mexico. Let's assume that all of her female descendants survive to produce offspring and that the fourth generation monarchs will overwinter in Mexico the following year. Use your function to determine the number of female descendants of this butterfly that will overwinter in Mexico the following year. Express your answer in billions.

4. In the winter of 2013-2014, an estimated 30 million monarchs were overwintering in Mexico. Let's assume half of these were female. Again, assume that the fourth generation descendants will overwinter in Mexico the following year, and that none of the descendants die before reproducing. If this were the case, how many monarchs would we expect to see overwintering in Mexico the following year? How many times larger is the population in 2014-2015 than the 2013-2014 population? Express all numerical values using scientific notation. Follow the steps below:

   Number of monarchs in winter 2013-2014 = $30\ million = 3 \times 10^7$

   Number of females in winter of 2013-2014 =

   Number of females in winter of 2014-2015 =

   Number of monarchs in winter of 2014-2015 =

   Number of times larger =

So, why aren't we seeing more monarchs?

The bar chart in Figure 1 on the following page shows the area of the region (in hectares) occupied by overwintering monarch butterflies in Mexico each winter between 1993 and 2015.

5. The monarchs cluster in huge numbers on the oyamel fir trees for protection and warmth. The density of monarchs within this region is thought to be as high as 50 million butterflies per hectare!

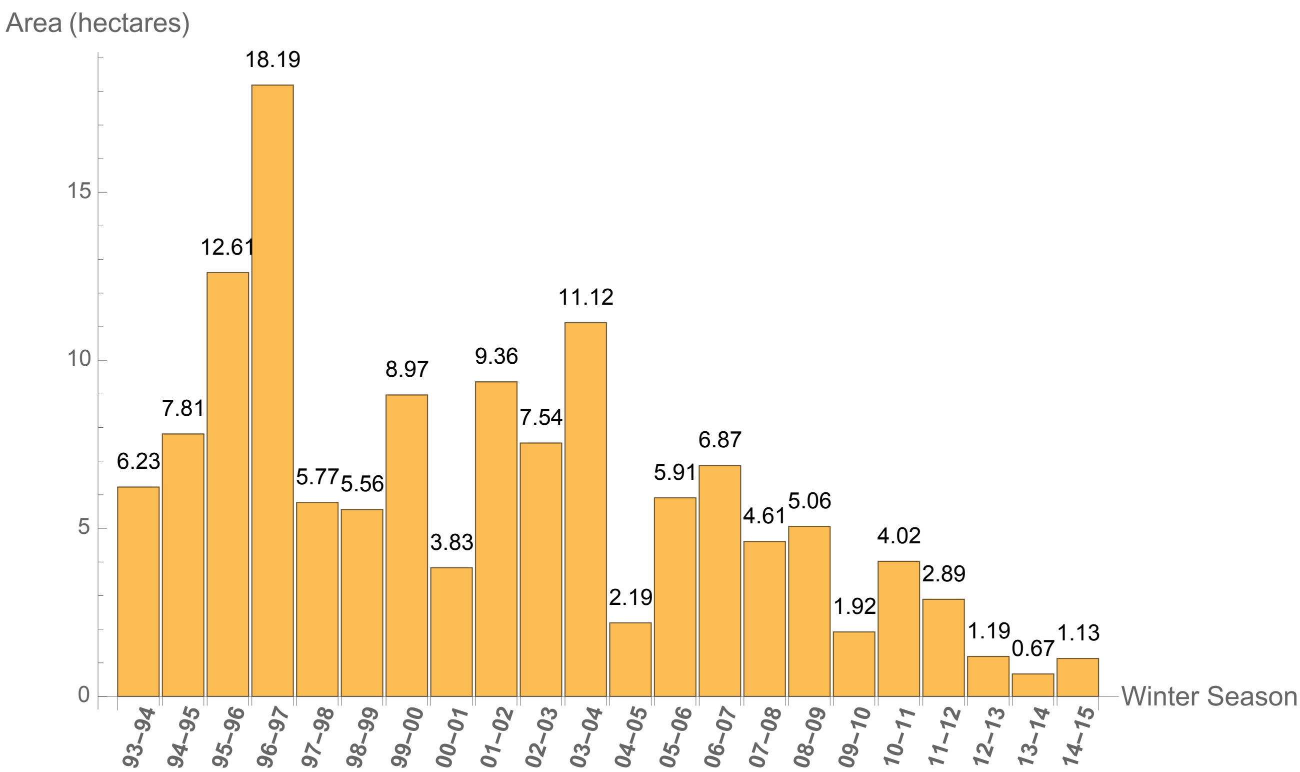
   a. Use this information and the bar chart in Figure 1 to complete the following table.

Overwintering Season	**1999-2000**	**2014-2015**
Estimated Monarch Population (in millions)		

b. Compare the 2014-2015 estimate in part (a) with the 2014-2015 estimate in exercise 4. What does your comparison suggest?

c. By what percentage has the estimated overwintering population decreased between the 1999-2000 and the 2014-2015 winter seasons? Recall that the percentage decrease in the overwintering population is found by subtracting the 1999-2000 and 2014-2015 populations and then dividing by the 1999-2000 population. Move the decimal point two places to the right to express as a percent.

Figure 1. Area occupied by overwintering monarchs in Mexico.



The 2014-2015 estimate in exercise 4 assumes that **all** female offspring survive to become adults, successfully migrate to favorable habitats, and successfully reproduce. This is generally not the case. Many monarchs do not survive to successfully reproduce.

The main factors contributing to monarch mortality include:

- Predation by birds
- Parasitism
- Habitat loss that makes it difficult to locate enough nectar to fuel migration and enough milkweed on which to lay eggs
- Unfavorable weather (for example, droughts or colder than average temperatures) that leads to unfavorable habitat or disruptions in the migration timing

Studying the cumulative impact of these factors on monarch survival is further complicated by the fact that these factors vary geographically and also change over time. For the remainder of this activity, we will explore one of these factors in one geographic region. We'll examine decreasing monarch habitat in the Midwest—Iowa in particular—and the potential impact on monarch populations.

What's happening to milkweed populations in Iowa?

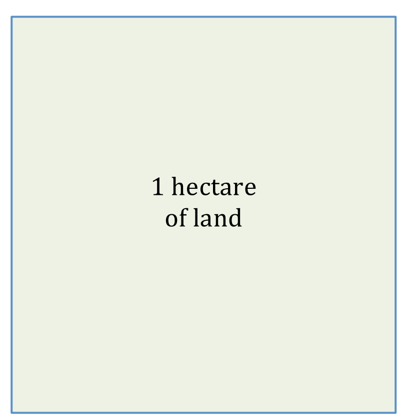
6. Table 1 below summarizes published findings from research on milkweed densities in Iowa. (Pleasants and Oberhauser, 2012). CRP land refers to previously farmed land that is currently enrolled in the federal Conservation Reserve Program.

Table 1. 1999 Average Iowa Milkweed Densities & Average Annual Change		
	1999 average milkweed density (m^2/hectare)	**Change in milkweed density during the period 1999-2010**
CRP land	212	5.2% average annual decrease
Pastures	14	5.2% average annual decrease
Roadside Ditches	99	No significant change
Corn & Soybean fields	23	14.2% average annual decrease

a. Consider the 1999 average milkweed densities shown in Table 1. When we say that the milkweed density in pastures is 14 m^2/hectare, we mean that one hectare of Iowa pasture contains an average of 14 square-meter patches containing milkweed. To visualize the densities in Table 1, imagine that the square below represents one hectare of CRP land, pasture, roadside land, or corn/soybean field. Recall that a one-hectare square is $100 \text{ m} \times 100 \text{ m} = 10{,}000 \text{ m}^2$. In separate corners of the square, draw four rectangles whose areas are approximately 212 m^2, 14 m^2, 99 m^2, and 23 m^2. (Remember that the area of a rectangle is equal to the length times the width. Here, just choose integer lengths and widths for the rectangles (in meters) so that the areas will be approximately what you want.)

These four rectangles represent the average areas containing milkweed plants within a hectare of CRP, pasture, roadside, or corn/soybean field in 1999. Label the rectangles "CRP", "pasture", "roadsides", "corn/soybean field" respectively for future reference.

What do you notice about the amount of milkweed growing within these different land types?



b. Now, consider the annual decreases in milkweed density in the last column of Table 1. Milkweed density in Iowa crop fields has decreased over time in response to increasing application of chemical herbicide. Milkweed density in CRP lands and pastures has similarly decreased, while roadside density has remained effectively unchanged. One possible reason for this is that milkweed tends to thrive in disturbed habitats. It is quick to establish itself after an area is mowed or plowed, but may ultimately be outcompeted by other plants over time. So, an area that is regularly mowed (for example, along roadsides) might sustain more milkweed, while an area that has been left undisturbed (for example, in CRP lands or pastures) may see less milkweed over time.

Determine the annual decay factors for milkweed density in Iowa CRP lands, pasture lands, and corn/soybean fields. Remember that an annual decay factor represents the percentage remaining after one year.

Annual decay factor for milkweed density in CRP lands and pastures =

Annual decay factor for milkweed density in corn/soybean fields =

Use your decay factors to complete the following table.

Year	1999	2000	2001	2010
Milkweed density in CRP lands (m^2/hectare)	212			
Milkweed density in pastures (m^2/hectare)	14			
Milkweed density in corn/soybean fields (m^2/hectare)	23			

c. Let t represent time (in years since 1999). So, for example, $t = 0$ represents the year 1999, $t = 1$ represents the year 2000, and so forth. Let M_{CRP} represent average milkweed density in CRP lands (in m^2/hectare). By observing the pattern in the calculations for part (b), write down an equation that will estimate the value of M_{CRP} at time t. Hint: The equation will have the form $M_{CRP} = b(R)^t$. You need to determine the values of b and R.

d. Again, let t represent time (in years since 1999). Let M_P represent average milkweed density in pastures (in m^2/hectare) and let M_{CS} represent average milkweed density in corn and soybean fields (in m^2/hectare). Using your work in part (b) again, write down equations that will estimate the values of M_P and M_{CS} at time t.

e. Suppose that milkweed density in Iowa CRP lands, pastures, and corn/soybean fields continues to decrease by respectively 5.2% and 14.2% annually even after 2010. In this case, what are the projected average milkweed densities currently? Make sure you include the units!

Table 2. Projected Current Average Iowa Milkweed Densities	
CRP land	
Pastures	
Roadside Ditches	
Corn & Soybean fields	

f. Go back to exercise 6 (a). Inside each rectangle that you drew previously, draw and shade another rectangle that represents the projected current milkweed density from part (e). What observations do you have?

g. So, how much milkweed is growing in Iowa? The state has roughly 700,000 hectares of land in the Conservation Reserve Program, 1,400,000 hectares of pastures, 400,000 hectares of roadside land, and 9,300,000 hectares of corn and soybean fields. Use the milkweed densities from Tables 1 and 2 to estimate the land areas containing milkweed in 1999 and in the current year. Express your answers in **millions of square meters**.

<table>
<tr><th colspan="4">Table 3. Land Area Containing Milkweed (millions m²)</th></tr>
<tr><th rowspan="2">Land Types</th><th rowspan="2">Total Land Area (hectares)</th><th colspan="2">Area containing milkweed (million m²)</th></tr>
<tr><td>1999</td><td>Current Projection</td></tr>
<tr><td>CRP lands</td><td>700,000</td><td></td><td></td></tr>
<tr><td>Pastures</td><td>1,400,000</td><td></td><td></td></tr>
<tr><td>Roadsides</td><td>400,000</td><td></td><td></td></tr>
<tr><td colspan="2">Non-crop land Total
(CRP+Pasture+Roadsides)</td><td></td><td></td></tr>
<tr><td>Cropland Total
(Corn and soybean fields)</td><td>9,300,000</td><td></td><td></td></tr>
</table>

So, what does milkweed abundance imply about monarch populations in Iowa?

The number of monarch eggs observed on a milkweed stem (called the *egg density*) has been shown to be about 3-5 times larger for milkweed growing in corn/soybean fields than for milkweed growing in non-crop lands (like CRP land, pastures, and roadsides). Egg densities vary throughout the summer, generally attaining peak (largest) values in May and August. In Iowa, peak egg density values recorded during the time period 2000-2003 ranged from 0.659 to 1.661 eggs/stem in corn/soybean fields and from 0.197 to 0.345 eggs/stem in non-crop lands (Pleasants and Oberhauser, 2012).

For the calculations in this activity, we'll make the following two assumptions:

- **<u>Assumption 1</u>**: We'll assume peak egg density values of 0.9 eggs/stem for corn/soybean fields and 0.2 eggs/stem for non-crop lands. (These are the average values over the years 2000-2003 from the Iowa study referenced above.)
- **<u>Assumption 2</u>**: We'll assume that in patches of land where milkweed is observed (corn/soybean fields and non-crop land), we will see about 4 milkweed stems per m^2.

7. Use these two assumptions to estimate the maximum number of monarch eggs per m^2 in corn/soybean fields and in non-crop fields.

   Maximum number of eggs/m^2 in corn/soybean fields =

   Maximum number of eggs/m^2 in non-crop lands =

8. Now, use the 1999 and current year estimates of land area containing milkweed from Table 3 and your answers from exercise 7 to obtain estimates for the number of monarch eggs laid at the peak of the reproductive season in each of these two years. **Express your estimates in millions of eggs.**

   Peak egg laying in 1999 =

   Projected peak egg laying in the current year =

   By what percentage has peak egg laying decreased since 1999?

Simply knowing the two Iowa egg estimates above is not enough to be able to estimate the adult population that will migrate from Iowa to Mexico. However, these egg estimates do give us a rough measure of the reproductive success of monarchs in Iowa in different years. How significant do you think the decrease in peak egg laying is, in terms of understanding monarch population trends over the last fifteen years?

9. (For Discussion) Summarize the key points of this activity. What more would you want to know to investigate the issue more fully?

Homework Exercises

1. Refer back to Table 3 in exercise 6 (g) for this exercise.
   a. Compare the total area containing milkweed in non-crop lands with the total area containing milkweed in corn/soybean fields. How do the areas compare in 1999? How do the areas compare in the current year?
   b. Rank the four land types (CRP, roadsides, pastures, and corn/soybean fields) from greatest area containing milkweed in 1999 to least area containing milkweed.
   c. Repeat part (b) for the current year.
   d. How has the ranking changed from 1999 to present?
   e. For each of the four land types, by what percentage has the estimated area containing milkweed declined since 1999?
   f. Which of the four land types has seen the greatest loss in milkweed since 1999?

2. In order to estimate peak egg laying in 1999 and the present year, we made an assumption (Assumption 2) that we would see 4 milkweed stems per square meter. What if we had assumed instead that we would see 2 milkweed stems per square meter. Which of your answers in exercise 8 is NOT affected?

3. In exercises 2-4 of the activity, we assumed that all monarchs survive to adulthood. However, mortality is thought to be quite high for monarch larva. Let's assume that only 1% of a female's eggs survive to adulthood.

   a. How many adult female offspring does each female produce now?
   b. Repeat exercise 4 of the activity with this change.
   c. How different are your results from part (b) compared with the estimates you obtained when you assumed no mortality.

4. **Do some research**. Find out information about citizen science opportunities related to monarch butterflies at the website http://www.monarchjointventure.org/get-involved/study-monarchs-citizen-science-opportunities/. Write a paragraph addressing the following questions: What and who are citizen scientists? What do citizen scientist volunteers with the Monarch Larva Monitoring Project (MLMP) actually do? Where is the MLMP organization based? What are the objectives of the MLMP program?

Acknowledgements

Special thanks are due to the many citizen scientists who provide data for the Monarch Larvae Monitoring Project. Data collected and recorded by these volunteers was used in the research study by J. Pleasants and K. Oberhauser (2012) referenced heavily in this activity.

Photos Used in this Activity

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Chapter 4. Wind Energy

This is a two-part module. Although Part A is intended as preparation for Part B, each part can be used separately. Part A, *How much energy does a wind turbine produce?,* works with multivariable functions (formulas) and would be appropriate for use in college algebra, pre-calculus, and math for liberal arts courses. Part B, *Wind Energy: What does it cost?*, emphasizes general quantitative skills and therefore could be used in any of the above courses or in an intermediate algebra course.

Instructor Notes for Part A: *How much energy does a wind turbine produce?*

Synopsis: This activity is divided into two sections. The first section provides an informal introduction to energy and power. Power is defined as the speed (rate) that energy is transferred or delivered. In situations where the power is constant, one can calculate the energy consumed or delivered over a given time period by multiplying the power by the time duration. Working through a series of exercises, students become acquainted with common units of measurement for energy and power such as joules, kilowatt-hours, and megawatts. They use their unit conversion skills to estimate energy consumption of appliances and wind energy over short periods of time when power is assumed to be constant.

The second section walks students through the derivation of a mathematical model for the electrical energy delivered by a wind turbine over short periods of time under the assumption of constant wind power. Electric power delivered by a turbine is assumed to be proportional to the wind power entering the circular region swept out by the turbine's rotor blades (which we refer to in the activity as the ``sweep region"). The wind power entering the sweep region is a function of the upstream wind speed and the air mass entering the sweep region per second. This air mass in turn is a function of the air density, upstream wind speed, and the length of the turbine blades. Back substituting these functions into the equation for electric power and then multiplying by a given time duration leads to a function for the electrical energy delivered. Students then apply their functions in particular situations.

Skills/knowledge Required: unit conversion, SI prefixes, derivation and evaluation of multivariable functions, reading information from graphs, area of a circle, volume of a circular cylinder

Vocabulary Used: energy, power, joules, watts, power coefficient, mechanical/electrical efficiency

Structuring In-Class and Out-of-Class Work: Students in the *Mathematics of Sustainability* course required approximately 1.5 hours in class working in groups of 3-4 to complete the first section of this activity. After completing the first section, students worked exercise 1 from the Homework Exercises at the end of the activity as a post-class assignment. In this exercise, students practice calculating the energy consumed by particular appliances over given time periods. The amount of in-class time required to complete the second section of this activity varied. Instructors choosing to lead their students through the derivation of the model for the delivered electrical energy from a wind turbine may complete the second section in 1.5-2 hours. When students in the *Mathematics of Sustainability* course worked through the derivation in small groups, this section of the activity required 2-2.5 hours. After completing this section of the activity, student worked a subset of the remaining Homework Exercises.

Additional Notes:

- Students should be encouraged to slow down and re-read exercises as they work through this activity.

- The concepts of energy and power can be challenging for students to grasp and retain. Earlier in the activity, when energy is measured in joules and power in joules per second, the distinction is easier for students to remember. However, later in the activity, when the units of energy are given in kilowatt-hours and power in kilowatts, students sometimes forget the distinction. So, repeatedly emphasizing that power represents the amount of energy delivered per unit time will be helpful.

- Because the first section of this activity introduces students to the concepts and calculations that they will need in the second section focusing on wind energy, students appreciate having their work in the first section checked. In the *Mathematics of Sustainability* course, students displayed their work on the white board for all of the exercises in the first section enabling the class to offer feedback and ask questions before proceeding to the second section.

- Particularly, toward the end of the second section, there is a tendency for students to forget how to calculate the electrical energy in units of kilowatt-hours or megawatt-hours. Reminding students of their work in the first section of the activity, particularly exercise 12, will be helpful.

Web Resources:

Site	url
Sustainable Energy--Without the Hot Air By David MacKay	http://www.withouthotair.com/

Instructor Notes for Part B: *Wind Energy: What does it cost?*

Synopsis: This part of the module begins by exploring the potential costs associated with the installation and operation of a 2-megawatt turbine over a twenty-five-year period. A significant factor impacting costs is financing of the installation. This activity considers both fixed rate financing through a bank as well as interest-free, government-subsidized bonds known as Clean Renewable Energy Bonds. Students explore annual costs both during and post repayment. The activity then considers revenue scenarios should the energy produced by the turbine be sold to a local utility company, from which students can then evaluate the profitability of commercial wind energy under multiple scenarios. Students also explore the impact of renewable electricity Production Tax Credits on the profitability of wind energy. The homework exercise at the end of the activity prompts students to do some research on the financing of community wind projects.

Skills/knowledge Required: unit conversion, SI prefixes, functions

Vocabulary Used: energy, joules, watts, revenue, profit, production tax credits

Structuring In-Class and Out-of-Class Work: In the *Mathematics of Sustainability* course, this activity has been used both as an in-class, small-group activity and also as a take-home activity completed by individual students. Both formats have worked well. In class, the activity required about 1.5 hours when students worked in groups of 3-4 students. Instructors wishing to derive the payment formula for fixed rate loans will need to allow another 0.5-1 hour for this. Regardless of whether students completed the activity in or out of class, the research question in the Homework Exercises was completed independently outside of class.

Additional Notes:
- Calculator errors are common in exercise 1 (c) in this activity and students are frequently unsure about their answers. Instructors might consider having students check their answers before proceeding.

- If students have not worked Part A of this module, they will likely have difficulty completing exercise 5 (a). In this situation, instructors might consider working this exercise as a class with the instructor leading.

Web Resources:

Site	**url**
Sustainable Energy--Without the Hot Air By David MacKay	http://www.withouthotair.com/
Windustry	http://www.windustry.org/
Wikipedia (Derivation of loan payment formula)	https://en.wikipedia.org/wiki/Fixed-rate_mortgage

Module 2A: How much energy does a wind turbine produce?

(WindImages, 2009)

Wind energy is a renewable source of energy that is being utilized increasingly in the United States. In this activity you will estimate the amounts of energy that wind turbines of different sizes may produce. Before talking specifically about wind turbines, however, we will do a brief tutorial on the basic ideas of **power** and **energy** and become acquainted with their units of measurement.

An Introduction to Energy and Power

Every month, most homeowners receive a bill from their local power company for the electricity used by the homeowner over the previous month. Shown below is a segment from an actual monthly electric bill:

Statement	
Previous Balance	116.63
Payments	116.63-
Adjustments	
Balance Forward	
1265 KWH @ .095700	121.06
FIXED CHARGE	32.70
POWER COST ADJUSTMENT	10.63-
SALES TAX	7.83
PROGRAM CHARGE	1.33
EXISTING CREDIT	0.67-
TOTAL	151.62

Discussion Question: Look at the bill above. Can you tell how much energy this homeowner used in the past month? What are the units of measurement for energy in the bill? Can you tell how many cents this homeowner is being charged for each unit of energy used? If you are a homeowner, do you know how many cents you are charged for each unit of (electrical) energy you use?

Thinking about energy and power might seem a little intimidating. It's easier to think about lengths, areas, volumes, or even masses, with units that we can picture. For example, we can picture in our minds what it means if someone can jump a distance of 10 feet. It means they're probably training for the long jump! But, it's harder to picture what it means if a household uses 1265 kilowatt·hours (abbreviated kWh) of energy. Is this household using a large amount of energy? In this case, we may not be sure. So, we're going to begin with an informal discussion of energy and power, trying to envision these concepts and their respective units.

1. Let's start with **energy**. **The joule (abbreviated J) is the basic unit of measurement for energy**. One joule is approximately the energy a person would expend to lift a small apple 1 meter against the Earth's gravity. That's not a lot of energy. But what if a person lifted a million small apples 1 meter. They would have expended one million joules of energy, or rather 1 megajoule of energy. (Remember that the prefix *mega* means one million.) Now that might seem like a lot of energy, but consider that setting a gallon of gasoline on fire will produce about 130 megajoules of heat energy. How many apples would a person need to lift to expend 130 megajoules of energy?

2. Let's talk about the energy in the wind. All moving objects have kinetic energy. An object having a mass of m kilograms (kg) moving at a speed of v meters/second will have kinetic energy E given by the function

$$E = \frac{1}{2}mv^2.$$

This energy is given in joules. So, how does this apply to wind? Well, air has mass. In fact, with the exception of the higher elevation mountainous areas of the U.S., a cubic meter of air has a mass of 1.225 kg. Wind is just a moving mass of air, and therefore it has kinetic energy that can be calculated using this function. Let's try this out. Suppose a wind gust containing 10 cubic meters of air is blowing at a speed of 15 miles per hour (which is equivalent to 6.7 meters per second). What is the total mass of this air, and how much kinetic energy is contained in this wind gust?

Note: The number **1.225 kg/meter^3 is referred to as the mass density of air**. To envision what this number means, imagine a cubic-shaped region of air with dimensions 1 meter × 1 meter × 1 meter. This region will contain about 1.225 kg of air.

3. Now let's talk about power. **Power is the speed at which energy is transferred or delivered**. Suppose that a wind gust strikes you over a period of 10 seconds, delivering 1000 J of its energy to you. How much energy (on average) did this wind gust deliver to you each second? In other words, what is the wind's power? Be sure to give units.

4. Now, suppose that the wind gust delivered 1000 J of energy to you over a period of 5 seconds. What is the wind's power now? Again, be sure to give units.

5. Which of the wind gusts in exercises 3 and 4 has the greatest power? Which wind gust would feel more powerful?

So, in summary, when more energy is delivered or transferred each second, it will be more powerful.

6. **The basic unit of measurement for power is the watt. One watt is equivalent to 1 joule of energy transferred per second**. Express your answers to exercises 3 and 4 in terms of watts.

7. (Discussion) A common mistake that people make is to mix up energy and power. Take a moment in your group to discuss the difference between energy and power. Then critique the following statement: "The power of an appliance is 100 watts per second." Explain why this statement does not make sense and how you could change it to make it correct.

8. Check your understanding of watts. What do you think it means for a light bulb to have a 100-watt rating? What does it mean for a microwave oven to have a 1500-watt rating?

9. Suppose that you have turned on a lamp for three hours. If the lamp uses a single 100-watt light bulb, how much energy has the lamp used? Give your answer in megajoules. Do this using the following steps.

    a. Express the 100-watt power in units of megajoules per hour using your unit conversion skills. Interpret your answer.

    b. Multiply the power (now given in units of megajoules per hour) by the three hours to calculate the total megajoules of energy used.

This last exercise illustrates that **energy is calculated by multiplying power by time when the power is constant (not varying)**. In the last exercise, you wanted the energy to have units of megajoules. Since time was measured in hours, you needed the power to have units of megajoules per hour. Why? Because

$$energy = \ power \cdot time = \frac{megajoules}{hour} \cdot hour = megajoules.$$

The last exercise and the next exercise both illustrate that **the units you choose for power and time determine the units of the energy.**

10. The energy bill on the first page of this activity showed that 1265 kilowatt·hours of energy were used by a household during a one-month billing period. What does this mean? Why isn't the energy measured in joules? As we stated earlier, joules are the basic unit of measurement for energy, but that doesn't mean that joules are the *only* unit of measurement for energy. A kilowatt·hour (abbreviated kwh) is another commonly used unit for energy. So, let's talk about it.

a. As we observed in the last exercise, energy is calculated by multiplying power by time and the units of the power and time determine the units of the energy. Rework exercise 9, but this time, express the power in kilowatts and leave the time in hours. Multiply the power and the time. What value do you obtain for the energy and what are the units?

You should arrive at 0.3 kilowatt·hours of energy. Your original answer was 1.08 megajoules. These two energy amounts are equivalent. The energy required to run a lamp for three hours appears relatively small when measured in kilowatt·hours, but huge when measured in joules. This is because a joule of energy is much smaller than a kilowatt·hour of energy. (This is just like saying a mile is much larger than an inch. If a person runs 2 miles, this is a relatively short distance when measured in miles, but appears huge when measured in inches.) In fact, using your unit conversion skills, you could show that **1 kilowatt·hour is equivalent to 3.6 megajoules of energy**.

b. Why do you think monthly energy bills are given in units of kilowatt·hours instead of joules? If you need a hint, the calculation below shows how 1265 kilowatt·hours is converted into joules.

$$\frac{1265\ kilowatt \cdot hours}{1} \cdot \frac{1\frac{joule}{second}}{1\ watt} \cdot \frac{10^3\ watts}{1\ kilowatt} \cdot \frac{60\ seconds}{1\ minute} \cdot \frac{60\ minutes}{1\ hour} = 4.554 \times 10^9\ joules$$

or 4.554 gigajoules.

c. Do you think wind energy would be best measured in joules or kilowatt·hours?

11. Suppose you are struck by a 50-watt air mass. What does this mean? Does this seem like a powerful wind gust? Hint: Think about the energy you would expend to lift a bag of 50 apples one meter against gravity over a period of one second.

12. Suppose wind strikes the blades of a turbine with a power of 1500 watts for a 2-hour period. How much energy does the wind deliver to the turbine blades?

    a. Calculate the energy in megajoules. Remember that energy is the product of power and time. Choose the appropriate units for power.

    b. Calculate the energy in kilowatt·hours.

Estimating the Electrical Energy Delivered by a Wind Turbine

Wind turbine systems convert the energy in the wind into electrical energy that we can use. The electrical energy delivered by a turbine depends on the amount of time the turbine is running and the speed at which it delivers energy (in other words, the power). Remember from earlier in this activity that if the power is constant over a given time period, the energy delivered by the turbine can be calculated by multiplying the power by the time.

13. Consider a turbine that delivers electrical energy at a constant rate of 0.6 megawatts.
    a. Express the 0.6 megawatt power in units of joules per second. What is this number telling you about the electrical energy delivered by the turbine?

    b. How many 1000-watt coffee makers could be powered using this turbine?

    c. How much electrical energy does the turbine deliver over a 3-hour period? Give your answer in megawatt·hours.

In this part of the activity, we will create a function to estimate the electrical energy delivered by a turbine when the power is constant. The function begins with what you already know. Namely,

$$\boldsymbol{E = P \cdot t}$$

Here, E represents the delivered electrical energy, P represents the power (the speed electrical energy is delivered), and t represents the time period. In the last exercise, $P = 0.6$ megawatts and $t = 3$ hours.

In practice, the value of P has to be calculated. Its value depends on several variables including the wind conditions in the turbine's vicinity and the efficiency of both the turbine and the system connecting the turbine to the power grid. So, in order to estimate P, we will need to introduce several new variables. The names and definitions of the relevant variables are given in the table on the following page for reference. Take a moment and read through these definitions before proceeding!

At each stage of the conversion from wind power to electrical power, energy is lost. So the electric power P delivered by the turbine will be less than the wind power P_{wind}. The power coefficient C and the turbine efficiency η represent the energy retained (not lost) in this process. Consequently,

$$\boldsymbol{P = \eta \cdot C \cdot P_{wind}}$$

Figure 1 on the following page provides a simplified illustration of this multistage process and respective energy losses. The diagram assumes that wind is blowing in one direction, namely directly at the turbine, making it possible to easily identify the air entering the sweep region each second. The diagram also shows the upstream and downstream wind speeds, v_{up} and v_{down}. Because a wind turbine obstructs the flow of air, you would generally expect that $v_{up} > v_{down}$.

Variable/Constant Name	Definition
$\boldsymbol{P}$	electric power delivered by the turbine (given in watts)
$\boldsymbol{P_{wind}}$	wind power entering the circular region swept out by the turbine blades which we'll call the "**sweep region**" (given in watts)
$\boldsymbol{v_{up}}$	upstream wind speed (given in meters per second)
$\boldsymbol{v_{down}}$	downstream wind speed (given in meters per second)
$\dot{m}$	mass of air entering sweep region each second (given in kilograms per second)
$\boldsymbol{C}$	referred to as the **power coefficient of the turbine**, represents the percentage of wind power entering the sweep region that is captured by the turbine (has no units)
$\boldsymbol{\eta}$	referred to as the **mechanical/electrical turbine efficiency**, represents the percentage of captured wind power that is delivered (has no units)

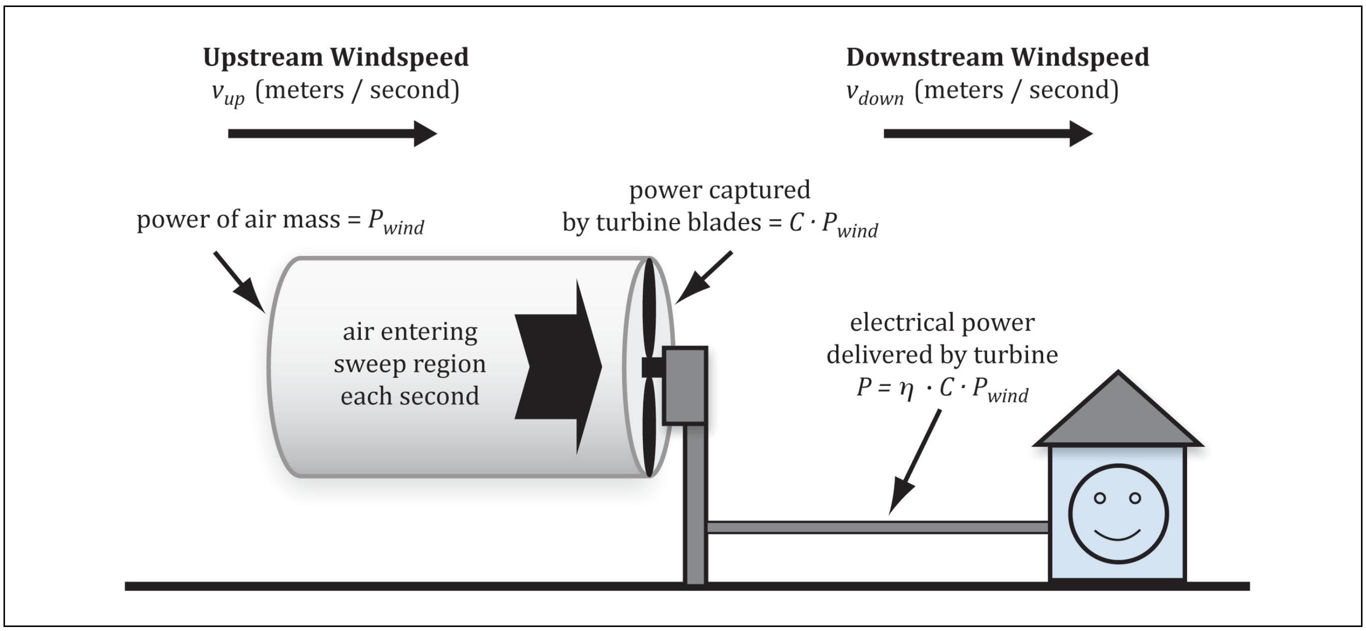


Figure 1: Simplified schematic showing the conversion of wind power into electrical power.

Let's estimate P_{wind}, the wind power entering the sweep region. Figure 1 shows the air entering the sweep region each second. We use the symbol $\dot{m}$ to represent the mass of this air. Since this air mass is upstream from the turbine, it is moving with speed v_{up}. From exercise 2 earlier in this activity, the energy in this air mass is $E = \frac{1}{2}\dot{m}(v_{up})^2$ joules. And since this is the energy entering the sweep region each second, the units are actually joules per second (or rather watts), which means that this quantity is actually the wind power entering the sweep region. In other words,

$$\boldsymbol{P_{wind} = \frac{1}{2}\dot{m}(v_{up})^2 \; watts}$$

14. Suppose that air upstream from a turbine is blowing at a speed of 15 miles per hour (which is equivalent to 6.7 meters per second), and 200 kg of air is entering the sweep region every second. Calculate P_{wind}, give its units, and explain in your own words what this number tells you about the energy entering the sweep region.

We're almost done! In the last example, the value of $\dot{m}$ was given to you. In practice, this too has to be calculated. So, let's see how this is done.

15. If the air is blowing in one direction, namely directly at the wind turbine, then all of the air that will eventually enter the sweep region lies in an invisible circular tube directly in front of the blades. Figure 2 illustrates this. To determine $\dot{m}$, the mass of the air entering the sweep region each second, we need to consider how fast the air is moving. For example, if the upstream wind speed is 6.7 meters per second, then all of the air in the tube located within 6.7 meters of the turbine would enter the sweep region in the first second. Similarly, all of the air located within the next 6.7-meter segment of the tube would enter the sweep region in the next second, and so forth. So, $\dot{m}$ is the mass of the air in each of these 6.7-meter segments of the tube.

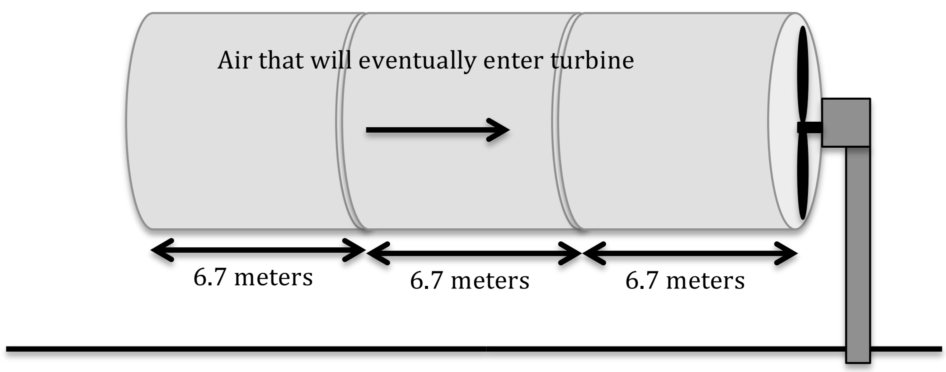


Figure 2.

To calculate $\dot{m}$, you will need the following information:

- The volume of a circular cylinder (in other words, a tube) is $V = hA$ where h represents the height of the cylinder and A represents the area of the base.
- The area of a circle is given by $A = \pi r^2$ where r represents the radius of the circle and π represents a value equal to 3.14159 (rounded to 5 decimal places).
- The mass density of air is 1.225 kg per meter^3. (Recall that we discussed this in exercise 2!)

a. Suppose that this turbine's blades are 80 meters in length. What is the area of the sweep region? Be sure to give the appropriate units. Round your answer to the nearest whole number.

b. Calculate the volume of one of the 6.7-meter segments of the tube. Be sure to give the appropriate units.

c. Calculate $\dot{m}$, the mass of the air in one of the 6.7-meter segments of the tube. Be sure to give the appropriate units.

d. Now that you've seen how $\dot{m}$ is calculated in a particular situation, use your work to help you create a function that will calculate $\dot{m}$ in similar situations. Assume that the length of a turbine's blades is l meters, the upstream wind speed is v_{up} meters per second, and the air density is 1.225 kg/meter^3. Write down an equation that will calculate $\dot{m}$.

$$\dot{m} =$$

16. Now it's time to put all the pieces together. Remember that our goal was to create a function that would estimate P, the electric power delivered by a turbine. We started with the equation

$$P = \eta \cdot C \cdot P_{wind}.$$

We then determined that

$$P_{wind} = \frac{1}{2}\dot{m}\left(v_{up}\right)^2.$$

Finally, in exercise 15, you created an equation that calculates $\dot{m}$. By performing a series of substitutions, write down a single equation that will calculate P. This equation will be a function where P is the output variable and η, C, l, and v_{up} are the input variables. Simplify your equation as much as you can and give the units of P. (Note that η and C do not have units. So, the units of P are the same as the units of P_{wind}.)

Recall that the electric energy E delivered by a turbine over a period of t hours is

$$E = P \cdot t$$

Using this equation, write down a function where E is the output variable and t, η, C, l, and v_{up} are the input variables. Give the units of E.

17. Let's try out the function above. Consider a turbine with 60-meter blades. Suppose that the wind upstream is traveling at a speed of 9 meters per second. Assume that the power coefficient of the turbine is 40%, while the turbine efficiency is 65%.

    a. Determine the electric power delivered by the turbine. Give your answer in megawatts.

    b. How many 1000-watt coffee makers could be powered using this turbine?

    c. If the power remains constant over a 4-hour period, how much energy does this turbine deliver? Give your answer in megawatt·hours.

18. A few last words about turbine power coefficients (C) and mechanical/electrical efficiencies (η):

    - Turbines generally have mechanical/electrical efficiencies (η) between 75-85%. So, for example, if a turbine has an efficiency of 75%, then $\eta = 0.75$.
    - It turns out that the value of a turbine power coefficient (C) depends on the ratio of upstream and downstream wind speeds. Let's call this ratio λ. (The symbol λ is the Greek letter lambda.) So, in other words, C is a function of the variable λ where $\lambda = \frac{v_{down}}{v_{up}}$. A graph of this function is shown in Figure 3 below.

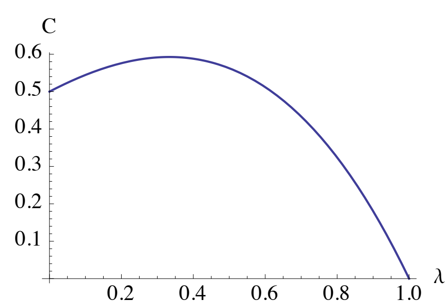


Figure 3.

    a. Let's try out the power coefficient graph above. Suppose that the wind upstream is traveling at a speed of 9 meters per second and the wind speed downstream from the turbine is 4 meters per second. Determine the value of λ and then use the value of λ to determine the power coefficient C.

b. According to the graph in Figure 3, what is the largest percentage of wind power capable of being captured by any turbine? Hint: Think about what the power coefficient C represents!

Homework Exercises

1. Explain in your own words what each of the following power ratings tells you about the amount of energy each appliance uses. Then, calculate the total energy consumed by running each of these appliances for the indicated amounts of time. Express your answers in both joules (J) and kilowatt · hours (kWh).

    a. Clothes washer: power rating 400 watts, operated for 40 minutes

    b. Hair dryer: power rating 1400 watts, operated for 5 minutes

    c. 16 cubic foot, frost-free refrigerator: 725 watts, operated for 24 hours

2. Consider a 75% efficient turbine with 50-meter blades, a constant upstream wind speed of 18 miles per hour, and a constant downstream wind speed of 15 miles per hour.

    a. Determine the area of the sweep region. Be sure to give the units.

    b. Use your unit conversion skills to convert the upstream and downstream wind speeds to units of meters per second. Note that there are approximately 1609 meters in one mile. For credit, you must demonstrate the process of unit canceling. To help you get started, note that you are trying to determine a quantity that has units of $\frac{\text{meters}}{\text{second}}$. So, you will begin your unit conversion by writing the following:

    $$\frac{\text{meters}}{\text{second}} = \frac{1609 \text{ meters}}{1 \text{ mile}} \cdot \quad (multiply\ by\ additional\ fractions)$$

    c. Determine the power coefficient for the turbine.

    d. Determine the power delivered by the turbine. Give your answer in kilowatts (kW).

    e. Determine the energy delivered by the turbine over a period of 4 hours. Give your answer in kWh.

    f. When in awake mode, a personal computer (CPU + monitor) consumes energy at a rate of 270 watts. How much energy does a single personal computer use over a period of 4 hours? Give your answer rounded to the nearest whole kWh.

    g. How many personal computers (in awake mode) can run using the energy generated from this turbine?

3. For this exercise, you'll need to refer back to the power coefficient graph in Figure 3.

    a. Turbines in use today have power coefficients in the range 25%-45%, considerably less than the theoretical maximum of 59%. Determine the values of λ that give power coefficients of 25% and 45%.

    b. Determine the λ-intercept and the C-intercept. In other words, give the value of λ when the graph intersects the λ-axis and give the value of C when the graph intersects the C-axis.

4. Care needs to be taken when estimating the energy generated by a turbine over long periods of time, like a year. In this activity, we worked exclusively with shorter time periods. Why not longer periods of time? Well, one of the central assumptions in this activity is that the power delivered by a turbine is constant. This would require wind speeds to remain constant. This assumption is easier to imagine if we restrict ourselves to shorter time periods.

   Adapting the functions you created in this activity to account for variable wind speeds requires higher-level mathematics. However, we can obtain rough estimates of energy generated over longer periods of time using the power ratings assigned to individual turbines. The power rating of a turbine is the power generated by the turbine when it is operating at full capacity. In other words, it is the maximum power that the turbine can generate. So, for example, a turbine with a rating of 2 megawatts will generate energy at a rate of 2 megajoules per second under optimal conditions.

   a. If a turbine with a power rating of 2 megawatts operates at full capacity for an entire year, determine the energy generated by this turbine. Give your answer in MWh. Note: You'll need to use your unit conversion skills here.

   b. On average, turbines operate at 30-45% of their full capacity. This means that the amount of energy actually generated is about 30-45% of the energy they generate when operating at full capacity. If the turbine in part (a) operates at 35% capacity, how much energy does it really generate over a year? Give your answer in MWh.

References

United States Department of Energy. (2014). *Estimating Appliance and Home Electronic Energy Use*. Retrieved from http://energy.gov/energysaver/articles/estimating-appliance-and-home-electronic-energy-use

Photos Used in this Activity

"Wind Turbines" by WindImages (http://flic.kr/p/7yjdkk/).

Module 2B: Wind Energy: What does it cost?

In this activity we'll assume that we are going to install and operate a 2-MW (megawatt) wind turbine for twenty years. To give you a feel for the energy output of such turbines in Minnesota, the city of Willmar (population ≈ 20,000) recently installed two such turbines to provide approximately 3% of the city's electricity demand. We'll explore several factors that impact the feasibility of such an initiative.

Financing the Installation

If a person is considering installing a wind turbine, they have to find a location, have the wind and soil conditions of the located site studied and assessed, acquire permits, install power lines and any additional infrastructure needed to connect the turbine to existing lines, purchase a turbine, the tower it sits on, and the base foundation, have it installed, and connect it to the local electrical grid. Each phase can be quite expensive. In fact, the total cost to install a utility-scale turbine in 2012 was between $1.3 million and $2.2 million per MW. For our 2-MW turbine, we'll assume that the total cost for installations is $4 million.

1. Let's assume, at least initially, that we will need to take out a loan to pay for this. Because a large amount is being financed, this is risky for a lending institution and they may ask to see our business plan, as well as the findings from pre-installation assessments and studies, before granting our loan. They may also require that we make an initial equity investment (in other words, we need to invest some of our own money before the lending agency will agree to give us a loan). We'll assume that our bank has agreed to allow us to finance (borrow) 70% of the $4 million. The remaining 30% must come from our initial equity investment. This will be a 10-year loan (10-15-year loans are common) carrying a fixed annual interest rate of 6% with payments made quarterly (4 times each year).

   a. Determine the size of the initial equity investment we need to make.

   b. Determine the remaining amount that we are financing.

   c. Determine the amount we will pay quarterly on this loan. You will need to use the function given below to calculate the loan payments:

   $$m \ = \ loan\ payment\ amount \ = \frac{\frac{r}{n}P}{1-\left(1+\frac{r}{n}\right)^{-nt}}$$

   Here, r represents the annual interest rate (in decimal form), P represents the amount financed, n represents the number of payments made annually, and t represents the duration of the loan (in years).

d. Determine the amount we will pay yearly on this loan.

e. Determine the total amount we will actually pay over the lifetime of the loan. Compare this to the amount that we financed (from part (b)).

2. Organizations such as municipalities, schools, and cooperatives can sometimes finance the installation of wind turbines using government bonds (called Clean Renewable Energy Bonds). The federal government pays the interest on these bonds in the form of tax credits. This provides interest-free financing for the organizations that qualify. Suppose we qualify for such a bond, payable over 15 years, with equal yearly payments. Let's assume that we are allowed to finance the entire $4 million. Determine the annual payments on this bond.

Post-Installation Costs

3. After the turbine is installed and connected to the local power grid, there will be additional costs. Annual operating and maintenance (O&M) costs are commonly estimated at 2-3% of the total cost of the project. We'll estimate our O&M costs to be 2.5% of the $4 million. Calculate our estimated yearly O&M costs.

4. (No work to do here, but you will need this info for later reference.) If we don't own the land where the turbine is located, we will have to rent the land from the owner. Land leases for large turbines like ours generally range between $3000 and $12,000 per turbine per year. Often, wind developers pay a fixed amount, say between $2500 and $4000, per MW installed on the land. Since our turbine is 2 MW, we'll use an estimate of $8000 annually to lease land.

5. In Minnesota, land devoted to wind energy production is exempt from property taxes. Instead, taxes on such property are structured according to the installed wind power on the property. This is called a production tax. Small-scale systems with installed capacities between 250 kW and 2 MW are taxed at a rate of $0.00012 per kWh of energy generated. Medium-scale systems with installed capacities between 2 MW and 12 MW are taxed at a rate of $0.00036 per kWh. Large-scale systems with installed capacities of 12 MW and higher are taxed at a rate of $0.0012 per kWh. Systems owned by political subdivisions that have installed capacities of 2 MW or less and residential systems with installed capacities under 250 KW are exempt from this tax.

   a. Let's assume that our turbine is expected to operate at 40% capacity on average. Given this, what is the estimated annual energy generated by our turbine? <u>Give your answer in kWh per year</u>.

b. Assuming we are not exempt from production taxes, our 2-MW installed capacity system falls on the border of two different tax tiers. Suppose we are taxed at the higher rate of $0.00036 per kWh of energy generated. Estimate our annual production tax.

c. Suppose instead that we are taxed at the lower rate of $0.00012 per kWh. Estimate our annual production tax now.

6. Wind turbines don't last forever. Eventually, there will come a time when we have a difficult decision to make: Should we replace or upgrade the turbine, or should we discontinue wind energy production? Our decision, whether to further invest in wind energy or abandon it, will be driven largely by financial considerations. The current cost for a 2-MW turbine and tower ranges between $2.2 and $3.4 million. If other equipment requires replacing or upgrading, there will be additional costs. If our expectation is to continue the project indefinitely, then we will want to set aside money each year to save up for eventual replacement and upgrading. Let's assume that the estimated cost to replace our current system will be $3 million. Wind energy systems are currently replaced after around 20 to 25 years. Let's plan to replace our turbine after 25 years. Suppose we start setting aside money after we've repaid our 10-year loan or our 15-year bond. Complete the following table:

	Amount to set aside annually for $3 million turbine replacement
15-year Replacement Fund (After 10-year loan is repaid)	
10-year Replacement Fund (After 15-year bond is repaid)	

7. If you purchase your electricity from a local utility, you are charged a certain amount of money for every kWh of energy that you use. In this exercise, you will calculate how much money we have spent for every kWh of energy produced from our turbine. To do this, you'll take the total amount of money we spend annually and divide by the total kWh of energy produced annually by the turbine. <u>Give your answer in cents/kWh rather than dollars/kWh.</u>

   There are many possible answers because our annual costs depend on many factors: how we pay for the installation, whether or not we need to pay a production tax, and whether or not we need to lease land. We'll consider seven different scenarios, given in the table on the following page. Scenarios 1-4 represent years while the loan or bond is being repaid, while scenarios 5-7 represent years after the loan or bond is repaid.

	Scenarios	Total Annual Cost (Dollars per year)	Cost per kWh (Cents per kWh)
1	Loan payments + O&M + land lease + \$0.00036/MW production tax		
2	Loan payments + O&M + land lease + \$0.00012/MW production tax		
3	Bond payment + O&M + land lease + \$0.00012/MW production tax		
4	Bond payment + O&M + land lease (exempt from production tax)		
5	O&M + land lease + \$0.00036/MW production tax + 15-year replacement fund		
6	O&M + land lease + \$0.00012/MW production tax + 15-year replacement fund		
7	O&M + land lease + \$0.00012/MW production tax + 10-year replacement fund		

8. What differences do you note when you compare the years when we're making loan/bond payments with years after the loans/bonds have been repaid?

Revenue and Profit

9. If we decide not to sell any of the energy produced by our turbine and use the energy for our own needs, then the table on the previous page gives a range of possible prices for our wind energy. For comparison purposes, look up the average retail price of electricity in your state from the U.S. Energy Information Administration's website at http://www.eia.gov/electricity/state/. How does the retail price in your state compare with the possible prices for our wind energy?

10. If we decide to sell some or all of the energy our turbine produces to a local public utility company, we can use the revenue to offset our costs and possibly earn a profit. In order to sell our energy we need to have a contract with the utility company specifying the selling price for our energy over the duration of the contract. Let's consider two possible selling prices: 5 cents per kWh and 6 cents per kWh. Let's assume that we agree to sell all of our energy, which you estimated in exercise 5 (a). Complete the following table:

	\$0.05 per kWh	\$0.06 per kWh
Annual Revenue (sales)		

11. Compare the annual revenues in the above table with the possible annual costs that you determined in exercise 7. Identify scenarios when we will break even or turn a profit, and scenarios when we won't.

12. Remember that if we earn income on the sale of our energy, we will likely pay taxes on this income, which will cut any profits even further. This small profit margin (profit per year) is a major reason why lending agencies can be reluctant to grant loans for wind production systems. To promote the development of wind energy and other renewable forms of energy, the federal government institutes financial incentives in the form of tax credits (called production tax credits, or PTCs). Currently, the PTC is 2.3 cents per kWh for wind energy produced <u>during the first 10 years of operation</u>. This credit can be added to our annual revenues above for 10 years. Assuming we qualify for the PTC, calculate the total tax credit we would receive annually.

13. Now, complete the following table:

	$0.05 per kWh	**$0.06 per kWh**
Annual Revenue (sales + PTC) *(up through year 10)*****		

Note: Only individuals meeting highly restrictive criteria qualify for the PTC, so we can't be sure that we'll qualify.

14. Compare the annual revenues in the above table with the possible annual costs that you determined in exercise 7. It is frequently said that federal tax credits are necessary in order to stimulate the growth of wind energy. Based on your estimates in this activity, would you agree with this statement?

Homework Exercises

1. Do some research on community wind projects in the upper Midwest. Read the 2009 report, *Lessons & Concepts for Advancing Community Wind,* published by The Minnesota Project. The URL for the report is: http://d3n8a8pro7vhmx.cloudfront.net/windustry/legacy_url/1588/Advancing-Community-Wind_Dec09.pdf?1421783488. Write one-paragraph responses to each of the following questions:
    a. What is a community wind project?
    b. What are some of the hurdles community wind projects face?
    c. How have some community wind projects successfully financed the installation of their turbines? In particular, what is the "Minnesota Flip" model?

References

NC Clean Energy Technology Center. (2014). *Database of State Incentives for Renewables and Efficiency: Renewable Energy Production Tax Credit*. Retrieved from http://www.dsireusa.org/

United States Energy Information Administration. (2014). *State Electricity Profiles*. Retrieved from http://www.eia.gov/electricity/state/

Windustry. Retrieved from http://www.windustry.org/

Chapter 5. GHG Emissions Associated with Automobile Fuels

This module develops students' mathematical modeling skills as they develop multivariable functions to estimate annual greenhouse gas emissions associated with cars. As such, the module is most appropriate for use in college algebra, pre-calculus, or math for liberal arts courses.

Synopsis: In this module, students create a mathematical model to estimate greenhouse gas emissions for gasoline and diesel-powered passenger cars. Using unit conversion skills, they estimate greenhouse gases emitted per gallon of fuel used, which depends on the type of fuel (gasoline, diesel, and ethanol are considered). Multiplying by the annual gallons of fuel consumed gives a model for total annual emissions. Unit conversion skills are used again to derive an equation for annual gallons of fuel used from the daily miles driven and the average fuel economy of the car. Students then apply their mathematical model to estimate annual emissions for gasoline-powered cars using E10 fuel and diesel-powered cars. Cars powered in part or in full using electricity are currently not included. The activity concludes with an exercise in which students scale up from the annual emissions of a typical passenger car to the annual emissions of all passenger cars in the country and express this total as a percentage of annual road transport emissions worldwide. They then calculate the U.S. population as a percentage of world population and then discuss possible implications of these two percentages.

Skills/knowledge Required: unit conversion, percents, back-of-the-envelope estimation, derivation and evaluation of multivariable functions

Vocabulary Used: tonnes, megajoules, greenhouse gas, carbon dioxide equivalent, energy content, biofuel, ethanol, diesel, fuel efficiency

Structuring In-Class and Out-of-Class Work: Students in the *Mathematics of Sustainability* course were assigned the first two pages of this module as a pre-class reading assignment. The remainder of the module was completed in class in about 1.5 hours with students working in groups of 3-4. There is an extensive set of Homework Exercises for this module for use as post-activity assignments.

Additional Notes:

- It is important that students determine correct values in exercise 4 as these values are used throughout the activity. Instructors might include a formative assessment or class discussion after exercise 5 for this purpose.

- The original version of this module was much longer. The module was scaled back by moving several non-essential (but interesting) exercises from the activity into the homework exercises. Consequently, there is an extensive collection of homework exercises for this module. Some of these exercises are quite lengthy. So instructors should consider assigning only a small subset. Note that exercise 1 is particularly helpful in summarizing the key findings of the module as it compares emissions among comparable cars using different fuels, but it is one of the longer exercises.

Web Resources:

Site	url
U.S. Energy Information Administration	http://www.eia.gov/
U.S. Environmental Protection Agency	http://www.epa.gov/climatechange/
Oakridge National Laboratory: Transportation Energy Data Book	http://cta.ornl.gov/data/index.shtml

Module 3: Does my car emit greenhouse gases?

(JT, 2014)

Have you ever used an online calculator to estimate amounts like the monthly mortgage payment on a loan you are considering, or the monthly car payment for a car you want to buy, or to calculate your personal carbon footprint? Online calculators require you to provide some necessary information (inputs), like the loan amount you are considering and the interest rate. Once you enter the required information, the online calculator works behind the scenes to produce an estimate for you (the output). Does this sound like a function? Well, it should, because online calculators are nothing more than functions! In this activity, you'll create a calculator to estimate annual greenhouse gas emissions from cars. Since the type of fuel we use in our cars can dramatically impact our emissions, this will be an important input that we examine.

Corn: From Food to Fuel

Corn has become one of the most important crops grown in the United States, and it is a big part of a typical person's diet. Corn is found in the sodas we drink, the sweeteners we use, our meat, and most processed foods that we eat. So, why is corn used in so many of our foods? Corn kernels have a lot of energy stored inside them, in the form of starch. Because the United States government provides financial support to farmers who grow corn (in the form of subsidy payments), the price of corn on the market is kept low. As a result, corn has become an inexpensive form of caloric energy that food producers have adopted to feed their cattle and to feed us. The blessing and curse of this corn-based food production system for us consumers is that the low price of corn means that foods produced using corn are relatively inexpensive in the grocery stores and easier to over-consume.

Because corn is an inexpensive energy source grown here in the U.S., it is also increasingly used for non-food-related purposes. In particular, it is processed into a liquid fuel, ethanol. In recent years, the government has mandated that ethanol be blended into gasoline before being sold for use in cars in order to reduce U.S. dependency on foreign sources of fuel.

So what are our fuel options?

Consumers have access to a number of fuel options, the most common being gasoline, varying gasoline/ethanol blended fuels, and diesel fuel.

Almost all modern gasoline-powered cars can also use gasoline that has been blended with 10% ethanol, commonly referred to as E10. Cars with specialized engines, referred to as flexible-fuel vehicles (FFV), can accommodate fuels with higher ethanol concentrations. One such fuel blend is E85, which contains 50-85% ethanol.

There are also diesel-powered cars. Diesel fuel, like gasoline, comes from crude oil. However, diesel engines don't use a spark plug to initiate the combustion of the fuel. Rather, the diesel engine compresses the fuel to such a high pressure (and therefore temperature), that the fuel ignites without the need of a spark plug. This higher temperature combustion produces more energy, which means that diesel cars can travel more miles per gallon of fuel used. Most diesel-powered cars can also use (petroleum) diesel fuel that has been blended with 5-20% biodiesel fuel, produced from renewable animal fats and vegetable oils (frequently from soybeans). In this activity, we will not consider blended diesel fuels.

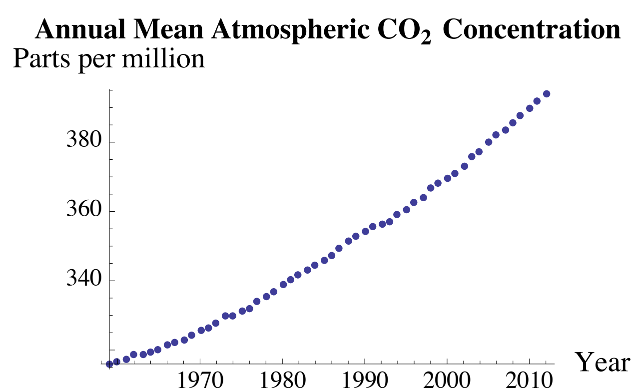
Increasingly common are electric-powered and gas-electric-powered (hybrid) cars. These cars, powered either in part or in full using electricity, rely on whatever fuel source is generating the electricity. Generally, this is natural gas and coal. In this activity, we will not consider greenhouse gas emissions associated from electricity use.

Discussion Question: If you or your family own a car, do you own a gasoline-powered, diesel-powered, gas/electric hybrid, or electric-powered car? What kind of fuel do you use? How do you decide which fuel to buy? Have you ever considered owning a car that requires a different fuel source?

Greenhouse Gases and the Earth's Climate System

How do the fuels we use in our cars produce greenhouse gas emissions, and why does it matter? Because our fuels all originate from carbon-based substances (also known as organic matter), the primary greenhouse gas that we need to consider is carbon dioxide. Through photosynthesis, plants absorb carbon dioxide from the atmosphere and store the carbon in their stems, leaves, and roots, which enables them to grow and thrive. When plants die or are cut down, some of this carbon is released back into the atmosphere, but a lot of plant carbon remains in the soil. When we burn fossil fuels (like gasoline, diesel, natural gas, and coal) we are releasing carbon that has been trapped in the ground for many thousands of years. On the other hand, when we burn biofuels like ethanol and biodiesel, we are releasing the carbon that was accumulating in a (corn or soybean) plant that we grew in one growing season. Another emission source associated with biofuels results from land use change. The plants used for production of biofuels would formerly have been used to grow food. So, additional land elsewhere will need to be converted into cropland for food production, and this conversion usually involves cutting down whatever plants and trees were growing on the land, thus releasing additional carbon dioxide back to the atmosphere.

Carbon dioxide (CO_2) is a naturally-occurring greenhouse gas in our atmosphere. Other naturally-occurring greenhouse gases include water vapor, methane, and nitrous oxide. These gases have the ability to absorb radiation from the sun, which warms our planet. This ``greenhouse effect'' is essential to most of the life on the planet, because without these gases, the average temperature of the planet would be about zero degrees Fahrenheit! However, greenhouse gas concentrations in the atmosphere have increased dramatically over the last century (see figure at right), generally believed to be the result of increased use of fossil fuels. This is of great concern to scientists and policy makers as they try to understand the many possible impacts this will have on the Earth's highly complex climate system upon which all life on the planet depends.



Source: National Oceanic and Atmospheric Administration (NOAA), Global Monitoring Division (See References for full citation.)

What are the greenhouse gas emissions from burning a gallon of fuel?

The table below displays energy and emissions values associated with the combustion of gasoline, (petroleum) diesel, and ethanol. The **energy content** refers to the amount of heat energy that is produced when a gallon of fuel undergoes combustion (in other words, when the fuel is burned). Since this combustion powers a car's engine, these fuels must produce a HUGE amount of energy. For this reason, we measure fuel energy content in units of megajoules (abbreviated MJ). Recall that the prefix mega means one million. So, a megajoule is one million joules of energy. To give you an idea of how much energy a megajoule is, consider that it takes about a joule of energy for a person to lift an apple 1 meter against gravity. So, a megajoule is the energy required to lift a million apples 1 meter against gravity. That's a lot of energy! For each megajoule of energy produced from fuel combustion, the corresponding greenhouse gas emissions (in grams) are given by the **Emissions Per Energy Generated**.

Fuel Type	**Emissions Per Energy Generated (grams CO_2e/MJ)**	**Energy Content (MJ/gallon) *Higher Heating Values**
Gasoline	93.08	131.2
(Petroleum) Diesel	91.944	146.1
Ethanol	63-87 (variable)	89.2

Note: You may be wondering about the units we are using for the greenhouse gases, namely "grams CO_2e". So, let's talk about that. Although carbon dioxide (CO_2) is the primary greenhouse gas we are considering, methane (CH_4) and nitrous oxide (N_2O) are also greenhouse gases that are associated with fuel combustion. In order to estimate total greenhouse gas emissions, it is common to convert the emitted grams of methane and nitrous oxide into an equivalent number of grams of carbon dioxide that would have the same global warming potential. When this type of conversion has been done, the combined greenhouse gas emissions are given in units of grams of carbon dioxide equivalents (abbreviated CO_2e).

1. Rank the above fuels in order from highest to lowest in greenhouse gas emissions per megajoule of energy generated by the fuel.

2. Rank the above fuels in order from the highest to lowest in energy content.

3. Suppose you ignite a gallon of gasoline on fire.
   a. How many megajoules of heat energy are produced?

   b. How many tonnes of greenhouse gases are released? Use your unit conversion skills here. Recall that there are 1000 kg in one tonne. Round your answer to the nearest thousandth.

4. Repeat exercise 3 for diesel fuel and ethanol. Fill in the following table. Round your values to the nearest thousandth. For ethanol, give both the low and high values.

Fuel Type	***E*** **Emissions Per Gallon (tonnes CO_2e/gallon)**
Gasoline	
(Petroleum) Diesel	
Ethanol	(give both low and high values)

5. Rank the above fuels from highest to lowest in greenhouse gas emissions per gallon of combusted fuel. Why is the ranking different than in exercise 1?

What are the yearly greenhouse gas emissions from driving a car?

6. Suppose that a person drives a diesel-powered car and they use 1000 gallons of diesel fuel annually. Estimate this person's annual greenhouse gas emissions. Use your work from exercise 4.

7. You can see from your work in exercise 4 that the greenhouse gas emissions that result from burning a gallon of fuel vary depending on the fuel type (gasoline, diesel, or ethanol). Let's let E represent a fuel's greenhouse gas emissions per gallon. Suppose that G gallons of fuel are needed to run a car for a year. Write down an equation that will calculate T, the total annual greenhouse gas emissions (in tonnes/year) from driving this car. You can use your work in exercise 5 to assist you.

8. E10 is a blended fuel consisting of 10% ethanol and 90% gasoline by volume. So, for example, a gallon of E10 fuel contains 0.1 gallons of ethanol and 0.9 gallons of pure gasoline. Suppose that a person drives a gasoline-powered car and uses 600 gallons of E10 fuel annually. Estimate this person's annual greenhouse gas emissions. For ethanol's emissions per gallon, use the average of the high and low values. Hint: How many gallons of ethanol are used annually, and how many gallons of gasoline are used?

9. So, in order to estimate the annual greenhouse gas emissions of a car, we need to know how many gallons of fuel the car uses each year. Most of us probably don't keep track of how many gallons of fuel we use in a year, but we can estimate this.

    a. For example, suppose a person drives an average of 38 miles each day and owns a car that has an average fuel efficiency of 23 miles per gallon. Use your unit conversion skills to estimate how many gallons of fuel this person uses in a year. (Note: Your answer here should relate back to exercise 8!)

    b. Now, use what you learned in the example above to create a general equation that will estimate G, the gallons of fuel used in a year. Let M represent the daily miles driven and let F represent the fuel efficiency of the car (in miles per gallon).

10. Suppose that a person drives a diesel-powered car having an average fuel efficiency of 42 miles per gallon. If the person drives an average of 60 miles per day, estimate the person's annual greenhouse gas emissions from driving the car.

11. We conclude this activity by considering a global perspective on the greenhouse gas emissions from our cars.

    a. There are an estimated 137 million passenger cars in the United States. Suppose all of them had annual emissions of approximately 6.8 tonnes/year. Estimate the total greenhouse gas emissions for all passenger cars in the country. Give your answer in millions of tonnes per year.

    b. In 2011, world greenhouse gas emissions associated with transportation were estimated at 5887.42 million tonnes CO_2e. Express the U.S. greenhouse gas emissions for passenger cars (from part (a)) as a percentage of the world greenhouse gas emissions associated with transportation.

    Note: When all transportation-related emissions are included (for example, large trucks, buses, air transport, maritime vessels), the U.S. emissions make up about 28% of the world's transportation-related emissions.

c. Also in 2011, the estimated population of the U.S. was 311.59 million while the population of the world reached 7 billion. Express the U.S. population as a percentage of the world's population.

d. Compare the percentages in parts (b) and (c). Discuss the implications.

Homework Exercises

1. Let's do a comparison of greenhouse gas emissions. Consider three cars with engines of similar size that are driven 40 miles per day with identical driving conditions.

   Car A: Gasoline-powered engine, using only gasoline, average fuel efficiency is 30 miles per gallon
   Car B: Gasoline-powered engine, using only E10 fuel, fuel efficiency is about 3% less than Car A
   Car C: Diesel-powered engine, using only (petroleum) diesel fuel, fuel efficiency is about 30% larger than Car A.

   Note: The reason that car B has a lower fuel efficiency than car A is that it is running on E10 fuel. Recall from exercise 2 in the activity that ethanol has a much lower energy content than gasoline. Since E10 is a mix of ethanol and gasoline, its energy content is greater than ethanol, but less than gasoline. Since a gallon of E10 generates less energy than a gallon of gasoline, a car using E10 fuel will not run as long as it could if it were using gasoline. In other words, the fuel efficiency of the car (miles driven using one gallon of fuel) will be lower when E10 fuel is used.

   Car C has a higher fuel efficiency than car A is because it is running on diesel fuel, which has a higher energy content than gasoline. Since a gallon of (petroleum) diesel generates more energy than a gallon of gasoline, a diesel-powered car can run longer than a comparable gasoline-powered car. In other words, the fuel efficiency of the diesel-powered car will be higher than a comparable gasoline-powered car.

   a. Rank these cars in order from highest greenhouse gas emissions to lowest emissions. Do this without doing any calculations. Just make an educated guess.
   b. Calculate the fuel efficiencies of cars B and C.
   c. Estimate the gallons of gasoline that car A will use, the gallons of E10 that car B will use, and the gallons of diesel fuel that car C will use over the course of a year.
   d. Estimate the annual greenhouse gas emissions for each of these cars in tonnes.
   e. Rank these cars in order from highest greenhouse gas emissions to lowest emissions according to your estimates in part (d).
   f. How does the ranking in part (e) compare to your earlier ranking in part (a)? Was your initial guess correct? If not, check to be sure that you didn't make any mistakes in parts (b-e). If you don't find any mistakes, go back through your calculations to determine why the ranking is different than you initially believed.
   g. Do a cost comparison of cars B and C. Go to a local gas station that sells diesel fuel and write down the cost per gallon for regular gasoline (which we'll assume is an E10 blend) and the cost per gallon for diesel. Use your unit conversion skills to estimate the average cost per mile for car B (using E10) and the average cost per mile for car C (using diesel fuel). How do the costs compare? Hint: You will need to use the fuel efficiencies of the two cars here!

2. How is a car's ***average*** fuel efficiency calculated? Cars have two different quoted fuel efficiencies, namely their city mpg (miles per gallon) and their highway mpg. A car's average fuel efficiency is a ***weighted average*** of these two efficiencies. To understand what we mean by a weighted average, let's consider an example.

   Consider a car that has city/highway fuel efficiencies of 24/28 miles per gallon. Suppose 50% of the miles driven by this car are on highways and the other 50% of miles driven are on streets. In this case, we would expect the average fuel efficiency to be exactly half way between 24 and 28, namely

26 miles per gallon. This is the value that you would have gotten by performing the calculation (24 +28)/2. Now, notice that

$$\frac{24+28}{2} = \frac{24}{2} + \frac{28}{2} = \frac{24}{1}\cdot\frac{1}{2} + \frac{28}{1}\cdot\frac{1}{2} = 24(0.5) + 28(0.5) = 24(50\%) + 28(50\%)$$

This calculation illustrates an alternative way of calculating averages. The 50% factors above come from the fact that 50% of miles driven were on streets and 50% were on highways. So, what if 60% of the miles driven were on streets and 40% were on highways? In this case, the average fuel efficiency is calculated as follows:

$$24(60\%) + 28(40\%) = 24(0.6) + 28(0.4) = 25.6\ miles\ per\ gallon$$

This average is a little lower than the earlier average fuel efficiency, reflecting the fact that more than half of the miles driven are on streets where the fuel efficiency of the car is lower. This average is referred to as a weighted average because the 24 carries greater "weight" than the 28 in determining the average.

a. Suppose a car has city/highway fuel efficiencies of 32/38 miles per gallon with 30% of the miles driven by the car are on city streets and 70% of the miles driven are on highways. What is the average fuel efficiency for this car?
b. What would your answer in part (a) be if 70% of the miles driven by the car are on city streets and 30% of the miles driven are on highways?
c. Your average fuel efficiency in (b) should be less than your average efficiency in (a). Explain why.

3. Estimate the annual greenhouse gas emissions for a gasoline or diesel-powered car you own, or for a car belonging to someone you know. (If your car is electric-powered, you will need to choose a different car for this exercise.)

   a. Write down the following information:
      - the model of the car,
      - whether it is gasoline or diesel powered,
      - whether it is a flexible-fuel vehicle (see description in part (b) below),
      - its quoted highway and city fuel efficiencies, (http://www.fueleconomy.gov/mpg/MPG.do?action=browseList),
      - the average number of miles the car is driven each day,
      - the percentage of miles driven on city streets versus highways.

   b. Determine the average fuel efficiency of the car. Refer to exercise 2 above for assistance here. A couple of important notes:

      - If your car is gasoline-powered and you purchase regular unleaded gasoline at gas stations, then you are probably using E10 fuel.

      - **Flexible-fuel vehicles** are much less common than other gasoline-powered cars. If you don't see an "FFV" or "FlexFuel" label on the body of your car near the gas tank, then it's not a flexible-fueled vehicle. Flexible-fuel vehicles are capable of using gasoline or any gasoline/ethanol blended fuel, up to 85% ethanol. Fuels that are at least 50% ethanol are referred to as E85. If your car happens to be a flexible-fuel vehicle, then you will need to consider the type of fuel you use most often, E10 or E85.

If you use E85, assume that about 65% of your fuel is ethanol and 35% is pure gasoline, and in this case you will need to reduce the quoted fuel efficiencies by 20% to account for the higher concentration of ethanol in the fuel.

(USDA, 2011)

c. Estimate the gallons of fuel used annually. If the car is gasoline-powered, again assume that E10 fuel is used. If the car is diesel-powered, assume that (petroleum) diesel fuel is used.
d. Estimate the annual greenhouse gas emissions for this car.

4. In this exercise, you will estimate the amount of agricultural land (specifically corn fields) required to produce the ethanol that your car in exercise 3 uses each year. If you don't use ethanol-based fuels in your car or you did not work exercise 3, use the following situation: a car is driven an average of 40 miles/day and has an average fuel efficiency of 30 miles/gallon using E10 fuel.

    a. Approximately how many gallons of ethanol fuel does your car use annually? If you worked exercise 3, then you should have already done this in exercise 3 (c).
    b. A single bushel of corn will produce about 2.6 gallons of ethanol. Corn yields in future years are expected to be around 183 bushels per acre. Using your estimate from 4(a), estimate the acres of agricultural land that will be needed to produce the ethanol that your car consumes annually. Use your unit conversion skills to obtain this back-of-the-envelope estimate.
    c. There are an estimated 140 million passenger cars in the U.S. Suppose all of these cars use the same amount of ethanol yearly as your car does. How many acres of corn fields will be necessary to produce the needed ethanol?
    d. What are some possible consequences of using agricultural land for production of fuel?

5. In this exercise, you will explore the relationship between fuel efficiency and gasoline consumption.

    a. Consider a person that drives 50 miles per day. Substitute 50 miles/day into the equation from exercise 9 (b) and simplify the equation. You now have a function where the output variable G represents the gallons of fuel consumed per year and the input variable F represents the fuel efficiency of the car.
    b. Complete the following input/output table for the function in part (a). Round your answers to the nearest whole gallon.

F (miles per gallon)	**15**	**25**	**35**	**45**	**55**
G (gallons per year)					

    c. Use the input/output pairs from part (b) to assist you in drawing a graph of this function. Draw as accurately as you can and be sure to label your axes appropriately!
    d. How does gasoline consumption change as the fuel efficiency of a car increases?
    e. Consider a car that has a fuel efficiency of 15 miles per gallon. Suppose the fuel efficiency of this car were increased by 10 miles per gallon. How much less gasoline would be used annually?
    f. Repeat part (e) for a different car that has a fuel efficiency of 35 miles per gallon.
    g. The gasoline savings in parts (e) and (f) are different despite the fact that you increased both cars' fuel efficiencies by the same amount! Use the graph you drew in part (c) to explain why your answer in (f) is smaller than your answer in (e).
    h. Your graph in part (c) should flatten out as the fuel efficiency grows. What are the implications of this trend for gasoline consumption?

6. **Do some research**. Ethanol is one example of a biofuel. What are biofuels? How are they different from fossil fuels? What are some other examples of biofuels, and what are these biofuels made from? What are some of the benefits and challenges of using biofuels to power cars? Write a one-page report summarizing your findings. Be sure to cite your sources.

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Photos Used in this Activity

JT. (2014). Tailpipe Exhaust [Photograph]. Retrieved from https://flic.kr/p/bXrmiA. No changes were made. License: https://creativecommons.org/licenses/by-nc-nd/2.0/

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Chapter 6. Carbon Footprints and Population Growth

This module explores the interplay between a linearly increasing U.S. population and linearly decreasing per capita carbon dioxide emissions over recent years, and the implications for the U.S. national carbon footprint. Familiarity with linear and quadratic functions is assumed. Therefore, this module is most appropriate for use in college algebra and pre-calculus courses.

Synopsis: This module explores trends in carbon dioxide emissions in the United States since 1990. Over this period, population growth in the country has been nearly linear. Per capita carbon dioxide emissions were fairly flat until around 2006 when they began to decrease. Students develop a mathematical model for total U.S. carbon dioxide emissions since 2013 using population data and per capita emissions data. They assume that population growth is linearly increasing, while per capita emissions are linearly decreasing. This leads to a downward-opening quadratic model for total carbon dioxide emissions. Students make projections using this model and compare with the 2020 projection from the U.S. Environmental Protection Agency. By drawing a rough sketch of the graph of the quadratic model, students better understand the projections yielded by the model, and reflect on appropriate use of the model for making projections. The activity concludes by considering likely trends in future population growth and per capita emissions, and the resulting trend in total emissions for the country.

Skills/knowledge Required: Unit conversion, percents, calculating and interpreting rates of change, deriving and evaluating linear functions, graphing lines, expressing quadratic functions in standard form, finding vertices, graphing parabolas

Vocabulary Introduced: Per capita, carbon footprint, projection

Structuring In-Class and Out-of-Class Work: Students in the *Mathematics of Sustainability* course at Metropolitan State University required approximately 2.5 hours, working in groups of 3-4 students, to complete the activity in class. Exercises 1-3 in this activity can easily be done as a pre-class assignment if desired. There is a research-related homework exercise at the end of the activity for use as a post-activity homework assignment.

Instructors wishing to minimize the in-class time required for this activity might consider splitting up the activity as a series of homework assignments over several days. For example, exercises 1-4 might comprise a first assignment; exercises 5-6 for a second assignment; and exercises 7-9 for a third assignment. Small blocks of class time could then be devoted to discussion and assessment.

Additional Notes:

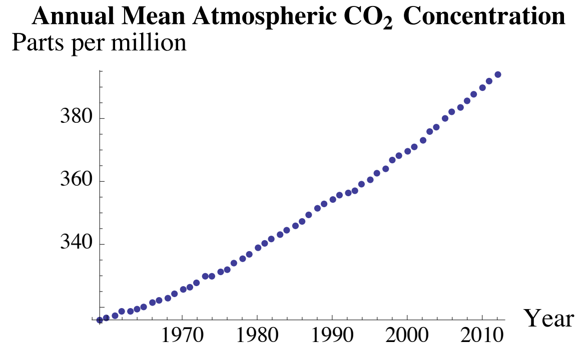
- In exercises 5 and 6, students derive equations that are used later in the activity. Instructors might consider having classroom discussion or other formative assessment after these exercises to enable students to correct mistakes and misconceptions.

Web Resources:

Site	url
2014 U.S. Climate Action Report	http://www.state.gov/e/oes/rls/rpts/car6/
United Nations	http://esa.un.org/wpp/Excel-Data/population.htm
Energy Information Administration	http://www.eia.gov/totalenergy/data/annual/index.cfm
National Oceanic & Atmospheric Administration (NOAA)	http://www.esrl.noaa.gov/gmd/ccgg/trends/

Module 4: Exploring Carbon Dioxide Emissions in the United States

Carbon dioxide (CO_2) is a naturally-occurring greenhouse gas in our atmosphere. Other naturally-occurring greenhouse gases include water vapor, methane, and nitrous oxide. These gases have the ability to absorb radiation from the sun, which warms our planet. This ``greenhouse effect'' is essential to most of the life on the planet, because without these gases, the average temperature of the planet would be about zero degrees Fahrenheit. However, greenhouse gas concentrations in the atmosphere have increased dramatically over the last century (see figure at right), generally believed to be the result of increased use of fossil fuels. This is of great concern to scientists and policy makers as they try to understand the many possible impacts this will have on the Earth's highly complex climate system upon which all life on the planet depends.

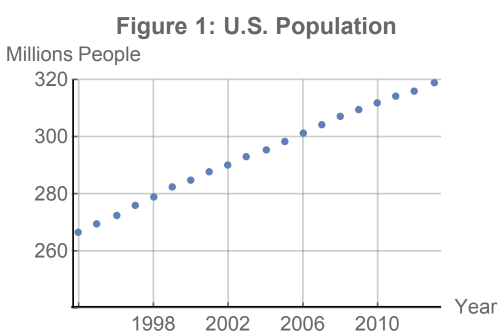


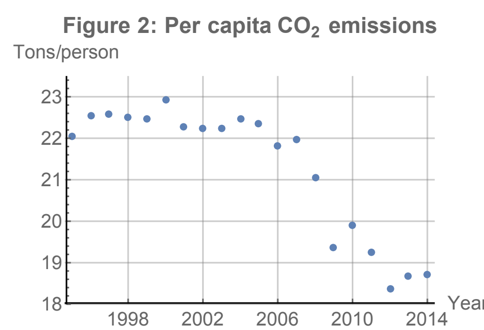
Source: National Oceanic and Atmospheric Administration (NOAA), Global Monitoring Division (See References for full citation.)

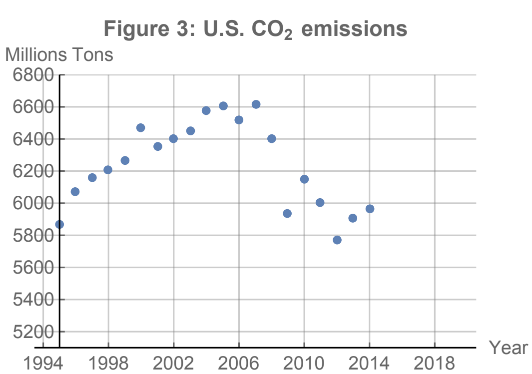
In this activity, we explore current and future carbon dioxide emissions from fossil fuel consumption in the United States.

Trends in Carbon Dioxide Emissions

Figures 1-3 below display estimated United States population growth, average annual CO_2 emissions per person in the United States, and total annual CO_2 emissions for the United States over the period from 1994 to 2014.







Sources: United States Census Bureau and Table 12.1: Carbon Dioxide Emissions From Energy Consumption by Source, December 2014 Monthly Energy Review, U.S. Energy Information Administration.

1. How would you use the data in Figures 1 and 2 to obtain the data in Figure 3?

2. Consider Figure 3. Why are CO_2 emissions generally growing during the period between 1994 and 2005? Why are emissions generally decreasing during the period between 2006 and 2012?

3. Figure 2 displays the average American's **annual carbon footprint** (in tons per year) during the period 1994-2014. What is the average American's annual carbon footprint in 2014? Perform a unit conversion to express your answer in pounds per day. This number represents the average American's **daily carbon footprint** in the year 2014. Note: There are 2000 pounds in 1 ton.

   2014 annual carbon footprint of an average American (tons/year):

   2014 daily carbon footprint of an average American (pounds/day):

Forecasting Future Carbon Dioxide Emissions

4. Figure 3 displays the total annual CO_2 emissions for the U.S. during the period 1994-2014. What are the emissions in 2005?

   Annual U.S. emissions in 2005 (million tons/year):

   In the 2014 U.S. Climate Action Report to the UN, the United States Environmental Protection Agency projects CO_2 emissions in the U.S. to decrease by approximately 7.6% between 2005 and 2020. If this happens, what will be the approximate emissions in the year 2020?

   EPA projected annual U.S. emissions in 2020 (million tons/year):

   Mark the projected 2020 emissions on Figure 3 with a point and label it "EPA" as a reminder that this is the projection from the Environmental Protection Agency. How does the projection fit with the trend in the data in recent years?

Let's determine a mathematical model, in particular a function, which will allow us to make our own projections about future CO_2 emissions in the United States. In order to create such a model, we will need to make assumptions about how the population in the U.S. is growing and how each person's emissions are changing over time. We will begin with an assumption about population growth.

Assumption 1: Population growth in the U.S. is approximately linear.

5. The population of the United States in the years 2006 and 2014 is estimated at 298.4 million and 318.9 million people respectively.

   a. Assuming population growth in the U.S. is approximately linear, calculate the annual rate of change of the population. Give your answer in millions of people per year.

   b. Use the estimate for the annual rate of change in part (a) to complete the table below.

Year	Years Since 2014	Projected Population (P)
2014	0	318.9 million
2015		
2020		

   Now, write down an equation that will project the size P of the U.S. population (in millions) t years after 2014.

   $P =$

   c. The equation in (b) graphs a line passing through the 2006 and 2014 population data points on Figure 1. Locate these data points. Using a straightedge, draw this line on Figure 1. Does the assumption of linear population growth appear reasonable?

Now that we have an equation for population growth, we turn our attention to how the annual emissions produced by an average person have changed over time in recent years.

6. Draw a line segment connecting the data points (2006, 21.8) and (2014, 18.7) in Figure 2.

   a. Does this line appear to fit the trend in per capita CO_2 emissions between 2006 and 2014?

Assumption 2: Per capita CO_2 emissions since 2006 are decreasing approximately linearly.

b. Determine the slope of the line segment you drew.

c. Explain what the slope in (b) tells you about the annual change in per capita carbon dioxide emissions. Your answer must include units.

d. Using the estimate for annual decrease in per capita emissions from parts (b) and (c), complete the following table. Show the units for the emissions values.

Year	Years Since 2014	Projected Per Capita CO_2 Emissions ($C_{percapita}$)
2014	0	18.7 tons/person
2015		
2020		

Now, write down an equation that will project $C_{percapita}$, the per capita carbon dioxide emissions, t years after 2014.

$$C_{percapita} =$$

7. Now that you have an equation describing population growth in the U.S. (from exercise 5) and an equation describing decreasing per capita carbon dioxide emissions over recent years (from exercise 6), you are ready to create a mathematical model (a function) that describes how the total U.S. carbon dioxide emissions change over time.

a. Using your two equations from exercises 5 (b) and 6 (d), determine an equation that will estimate C_{total}, the total carbon dioxide emissions for the U.S., t years after 2014.

$$C_{total} = C_{percapita} \times P =$$

b. What type of function is represented by this equation? (Linear, quadratic, exponential, logarithmic,…)

c. Use your equation to complete the following table. Round the emission values to the nearest million tons.

Year	Years Since 2014	Projected U.S. CO_2 Emissions (C_{total})
2014	0	
2016		
2020		

d. Mark the projected emission value for 2016 on Figure 3 with an “X”. Does this projection seem to fit the recent trend in U.S. emissions?

e. Mark the 2020 projected emissions on Figure 3 with a point and label it "Quadratic Projection".

f. Which 2020 projection, from the EPA or from our quadratic model, seems to best fit the trend in the data? Or, do both projections appear reasonable?

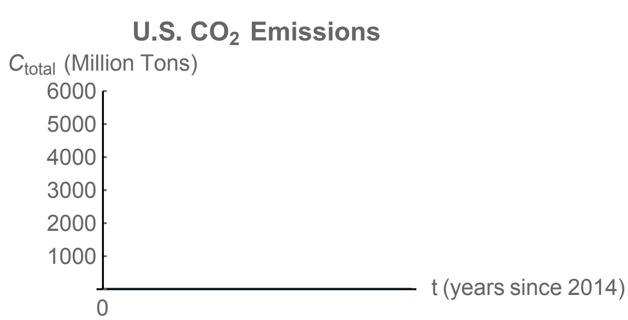
8. Let's use what we know about quadratic functions to better understand the projections we're obtaining from our quadratic model for U.S. emissions.

    a. Express the quadratic function for C_{total} in standard form. (In other words, express the equation in the form $y = ax^2 + bx + c$.)

    b. Will the parabola graphed by this function open upward or downward?

    c. Where is the vertex of this parabola located?

    d. Draw a rough sketch of this parabola for $t \geq 0$. You can use your calculations from exercise 7 (c) and your work above to assist you.



    e. According to the quadratic model, carbon dioxide emissions in the U.S. decrease indefinitely in the future. Does this seem plausible? Hint: Consider what an indefinite decrease in U.S. emissions would mean for the size of the U.S. population and/or per capita emissions.

f. When do you think it would be reasonable to use the quadratic model to estimate carbon dioxide emissions? When do you think it is probably NOT reasonable?

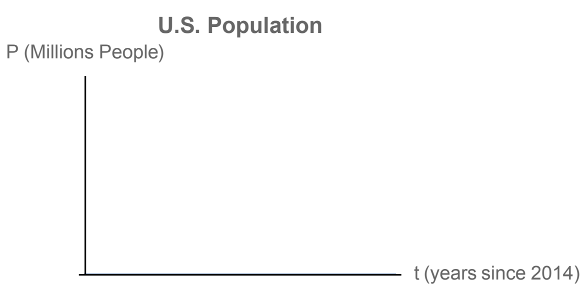
9. Let's reflect on the two assumptions that led to our quadratic model.

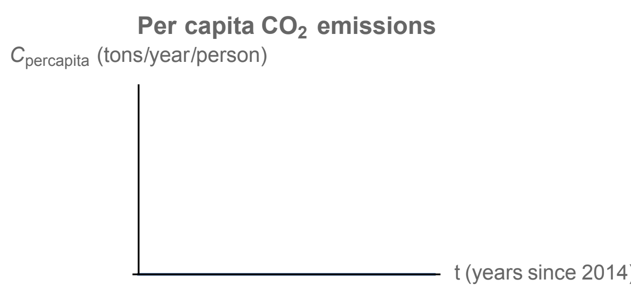
- **Assumption 1**: *Population growth in the U.S. is approximately linear.*
- **Assumption 2:** *Per capita CO_2 emissions since 2006 are decreasing linearly.*

a. Consider Assumption 1. Is this assumption likely to become implausible at some point in the future?

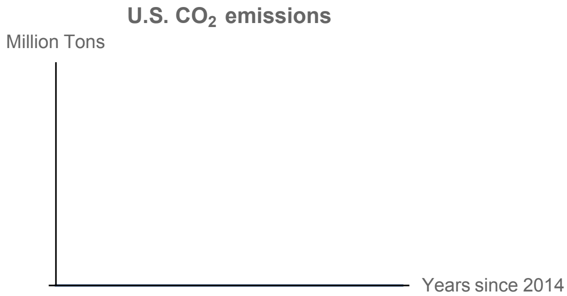
b. Consider Assumption 2. Is this assumption likely to become implausible at some point in the future?

c. What trends do you think would be more likely than linear growth for the U.S. population and linear decline for per capita CO_2 emissions after 2014? Give your answers in one or two sentences. Draw graphs to supplement your answers. Note: Your graphs should take recent trends into consideration.

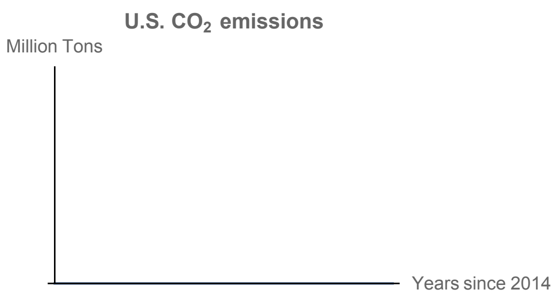




d. Suppose per capita emissions follow the trend that you graphed in (c), and the population of the U.S. continues to increase linearly (Assumption 1). Draw a graph showing the resulting trend in total CO_2 emissions for the country.



e. Suppose BOTH per capita emissions AND U.S. population growth follow the trends that you graphed in (c). Draw a graph showing the resulting trend in total CO_2 emissions for the country.



f. Look at your graphs in parts (d) and (e). Does the 2020 EPA projection seem plausible in either of these scenarios? Explain your answer in complete sentences.

Homework Exercises

1. **Do some research**. Access the 2014 U.S. Climate Action Report at www.state.gov/e/oes/rls/rpts/car6/. Use Chapters 5 and 6 of this report to assist you in responding to the following questions.

    a. What reasons are given for the recent decrease in U.S. carbon dioxide emissions? What is expected to happen to greenhouse gas emissions in the future? Write a one-paragraph response.
    b. Consider three different regions of the United States. In what ways is climate change impacting these regions? Write a one-paragraph response for each region.

c. Discuss two actions that are being taken by the federal government to address the impacts of climate change in the U.S. Write a paragraph response for each action.

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