J. M. Zobitz, T. Bibelnieks, M. Lester

International Sustainability Projects for Calculus

This teaching manual was created for Engaging Mathematics with support from the National Science Foundation.

An initiative of the National Center for Science and Civic Engagement, Engaging Mathematics applies the well-established SENCER method to college level mathematics courses, with the goal of using civic issues to make math more relevant to students. Engaging Mathematics will: (1) develop and deliver enhanced and new mathematics courses and course modules that engage students through meaningful civic applications, (2) draw upon state-of-the-art curriculum in mathematics, already developed through federal and other support programs, to complement and broaden the impact of the SENCER approach to course design, (3) create a wider community of mathematics scholars within SENCER capable of implementing and sustaining curricular reforms, (4) broaden project impacts beyond SENCER by offering national dissemination through workshops, online webinars, publications, presentations at local, regional, and national venues, and catalytic site visits, and (5) develop assessment tools to monitor students' perceptions of the usefulness of mathematics, interest and confidence in doing mathematics, growth in knowledge content, and ability to apply mathematics to better understand complex civic issues.

Updates and resources developed throughout the initiative will be available online at www.engagingmathematics.net. Follow the initiative on Twitter: @MathEngaging

This manual was typeset in LaTeX using the class tufts-latex: tufte-latex.github.io/tufte-latex/

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Acknowledgments

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Introduction

How do rural coffee farmers in Central America define and implement economic and environmental sustainability? If this sustainability is not achieved, what are the social justice issues for the farmers and their families? In this teaching manual we address these questions by investigating the multi-faceted aspects of sustainability of a rural farming cooperative in Central America through analysis of resource allocation, carbon footprints, and exploration of the long-term sustainability for ecotourism development.

The content in this teaching manual aims to to infuse more international perspectives into the mathematics curriculum motivated by two factors. First, mathematics (and other natural science) programs are very sequenced with a prescribed progression from introductory to advanced coursework. Augsburg College is a private liberal arts college located in Minneapolis, Minnesota. The calculus sequence is a standard three-semester set of courses covering single variable differential, single variable integral, and multivariable calculus. This sequencing, while historical and necessary, is a barrier for students interested in engaging in international education and coursework integrated into long or short-term study abroad programs. Student schedules become more constrained by upper level specialized courses. Related to this, presenting mathematics in realistic - and international - contexts *early* in the first few semesters of the mathematics major provides motivation for advanced study of mathematics as well as planning for (additional) extended international opportunities early in a student's college career. Second, this teaching manual addresses goals for Mathematics programs listed in the 2015 CUPM curriculum guide[1], specifically:

[1] Carol Schumacher, Martha Siegel, and Paul Zorn. *2015 CUPM Curriculum Guide to Majors in the Mathematical Sciences*. Mathematical Association of America, 2015. ISBN 978-0-88385-917-9

- **Cognitive Recommendation 1:** Students should develop effective thinking and communication skills.
- **Cognitive Recommendation 2:** Students should learn to link applications and theory.

- **Cognitive Recommendation 4:** Students should develop mathematical independence and experience open-ended inquiry.

- **Content Recommendation 3:** Mathematical sciences major programs should include concepts and methods from data analysis, computing, and mathematical modeling.

We see value in aligning the teaching manual content with the CUPM Curriculum guide recommendations. We hope that the projects presented in this manual will serve as an entry point to further advanced study and motivation of mathematical modeling.

The educational goals of the specific modules presented include constructing a mathematical model from intuition, incorporating eyewitness knowledge, and analyzing data (student-collected or cooperative provided). For each module we have intentionally limited the background content so that students have a sense of exploration and discovery into the process. Additional resources are noted for further information. There are several advantages to this discovery based approach that benefit students, instructors, and the cooperative. Both the students and the cooperative are equal partners in the learning process, creating for more opportunities for reciprocal understanding[2]. We believe that students are empowered to continue their study of mathematics and also interested in connecting their studies with international education. The cooperative has a richer context to explore and investigate different options pertinent to their future economic sustainability and success. Finally, as instructors we are enriched by the connections and cross-cultural engagement and learning facilitated through this module.

[2] Peter Levine and Karol Edward Soltan, editors. *Civic Studies*. The Civic Series. January 2014

The teaching manual was developed through a partnership with the GARBO Coffee Cooperative, located in Peñas Blancas in northern Nicaragua. This area has beautiful white cliffs that are surrounded by local, family-owned coffee farms, but are relatively unknown to tourists. In the year 2000 the cooperative was formed to gain better access to the coffee market. In the 2011-12 cycle, this cooperative produced 86,725 pounds of export-grade quality coffee for the world market. They have certification from FLO-Cert, one of the fair trade certifiers in Latin America.

This teaching manual describes two different modules that can be used in most calculus classes within the course of three to four weeks. The mathematical content is accessible to students in a first year calculus or applied calculus sequence. Students should have some knowledge of function families and skills developed in precalculus. Topics include rates of change, optimization, and modeling.

Format of the teaching manual

This teaching manual was prepared to provide an overview of the project, describing the initial setup as well as any explanatory notes or supporting documentation annotated in the margin. The appendices provided give additional information and instruction on accessing data. Given that technology changes over time, the instructions were current as of the time of the writing in late 2015. Specific questions can be directed to authors of this teaching manual.

Delivery of a module occurs over the course of six weeks in a typical semester as a series of three different investigative tasks. This type of module may be the first time that students use mathematics outside of traditional textbook contexts. Instructors considering adaptation to fit their learning setting should begin with the first activity that requires students to study and read about the place and context and form their initial impressions from an internet search or links provided in Appendix A. We prefer to give students specific instructions for each task.

If the module is given as a complete project, topics such as rates of change and their interpretation, differentiation, and optimization using differential calculus should already be familiar to students. Alternatively, smaller pieces of the chapter could be used as a motivating example for discussion of specific mathematical topics. Implementation of the module in this way provides a unified theme and story between the different mathematical ideas developed over a traditional semester of calculus.

Economic sustainability analysis of a rural Nicaraguan coffee cooperative

Introduction & Fact Finding about the Cooperative

THE INTRODUCTION TO this module sets the context for the mathematical investigations of different projects relating to (1) modeling and parameter estimation, (2) application of related rates, and (3) optimization. Typically this section requires minimal instructor preparation. Students are given an introduction to the coffee cooperative by the instructor, including showing short videos produced by a student group that traveled to the region in previous years as well as internet links for future exploration (see Appendix A).

The cooperative would like to expand its outreach to protect themselves from large price and demand fluctuations in the global coffee market and achieve economic sustainability. One option the cooperative is exploring for achieving sustainability objectives is farm-based eco-tourism. The cooperative strongly believes that economic diversification and prosperity can occur without sacrificing the natural beauty of the location or adversely affecting the coffee crop. The primary responsibility of guest accommodations would be the women and youth, allowing both groups to significantly contribute to the cooperative's economic livelihood.

The goal of this initial investigation to give students a fundamental understanding of the coffee cooperative's geographic and socioeconomic background. Guiding questions provide motivation for student investigation to ensure students fully engage with the project and take ownership of the tasks that will follow. At the completion of the initial investigation students should understand the scope of the project and the cooperative itself. The basic revenue model can now be introduced.

Revenue Model

A basic model for revenue production R (where the units of R are dollars per day) is the following:

$$R = p \cdot v - C, \tag{1}$$

where we have the following variables:

$R =$ revenue / day ($ / day)
$p =$ price or visitor charge ($) to stay at the cooperative
$v =$ the number of daily visitors (visitors / day)
$C =$ the cost of the cooperative ($ / day) to accommodate visitors.

The revenue model is purposely introduced without traditional function notation to emphasize that the variables, p, v, C, represent a quantity that can change or, mathematically, be considered as time-dependent. The definition of revenue as a quantity with units of dollars per day allows the model to be investigated later in the module as a derivative or differential equation. Later we will change the monetary unit to cordobas, the local currency in Nicaragua, but at this point it is helpful to utilize the US dollar for familiarity.

The first exploration of the revenue model focuses on how changes in the quantities p, v, and C affect changes in daily revenue, the output of the model. Questions that we have students investigate are the following:[3]

[3] The following questions can serve well as a separate homework assignment and basic interpretation of a revenue model, with positive and negative revenue. It may be helpful to have students understand that revenue is the *difference* between profit and costs.

1. Use several different values for p, v, and C to calculate revenue using the model. Interpret your results in each case.

2. Examine Equation 1 to determine an expression for the basic minimum price, p_{min}, the cooperative should charge to be profitable (i.e. $R > 0$). If the values of v and/or C change, how does the minimum profitable price change? Consider increasing or decreasing v and C independently and at the same time to answer this question.

3. Now examine Equation 1 to determine the minimum number of visitors, v_{min}, the cooperative must serve on a given day to be profitable. If both the price p and costs C change in value, what is the effect on the required minimum number of visitors for the cooperative to be profitable?

4. Assuming the price of the visitors is fixed, discuss with your group no less than three strategies that the cooperative can pursue to potentially boost revenue. Evaluate the three strategies on their merit, and determine appropriate data and metrics in your

evaluation. With your group, decide on the optimal strategy that you would recommend moving ahead with, providing justification in your report to the cooperative.

Refining costs and parameter estimation

The second part extends student understanding of the revenue model with the specific context of the GARBO cooperative. Students need to determine the currency conversion between the Nicaraguan currency of cordobas and their own local currency. We provide students with a table of common goods and their costs (see Appendix A). These items were generated by the cooperative from their experience hosting visitors. [4]

A revised revenue model is now introduced to the students that modifies the basic revenue model in Equation 1 that separates the visitor costs from the fixed costs, C:

$$R = p \cdot v - k \cdot v - C, \tag{2}$$

where k = visitor cost (cordobas / visitor) to accommodate guests. The advantage to Equation 2 is that it can be analyzed to include costs for items affected by external factors such as season, availability of goods, or the cooperative's ability to travel to purchase goods.

The focus of student work for analyzing Equation 2 is to estimate the daily cost (in cordobas) to accommodate visitors. Students are asked to plan out typical meal requirements for a given guest and create an itemized list. Students are encouraged to include items they think are important that are not listed in the table, but are required to provide a justified reasonable estimate of price(s). Students are then asked to identify costs that would be (1) dependent on each visitor and (2) costs that would be fixed or visitor *independent*. Finally, students answer a series of guiding questions to relate their work back to Equation 2, focusing on the parameters k and C:

1. Assuming the values of p, k, and C are known, what is an algebraic expression for the minimum number of visitors needed for the cooperative to be profitable? How does changing the parameters p, k, and C affect the number of visitors?

2. Examine your algebraic expression from the original revenue model. What is a necessary condition for the value of p?

3. Estimate a numerical value of C based on your itemized lists of fixed and variable costs. As you could imagine, the value of the parameter C depends on which items you focused on as visitor dependent. Provide justification for your reasoning.

[4] At this point a reflective assignment or group discussion could be added to the module. What food and consumable goods are readily available and which are cost prohibitive? How might this negatively affect the opportunity for ecotourism or how might it be leveraged to increase interest by visitors? The table given to students is a true reflection of the simplicity of life in the cooperative. For example, carbonated beverages, red meat, many common vegetables, breads and cereals are not listed in the table. Students could be asked to consider their own diets and use of consumable goods, reflecting on the differences of their life circumstances from the cooperative.

4. Estimate a numerical value of k based on your itemized lists of fixed and variable costs. To estimate k, it might be useful to take an accounting of the per item cost of goods, estimate the number of items that each visitor would use (for every day or for a visit), and then multiply the two values together. From that work, determine if a summative, or an average, cost per visitor is worthwhile.

5. Now that you have determined C and k, what is a necessary condition for p so that the cooperative is profitable ($R > 0$)? Would that price p be reasonable to charge cooperative visitors?

6. A recommendation from an external group is to give a daily wage to each family to offset the cost of receiving visitors. The wage should incorporate the fixed cost used to accommodate guests, and also any (human) time spent in the receiving of guests. If the cooperative will implement this plan, what type of information would be useful in setting a wage? What would be the wage you would recommend? Based on your investigations, determine how this wage will affect p. Would that price p be reasonable to charge cooperative visitors?[5]

An extension to this part is to have students develop reasonable formulas for possible periodic functions that would represent seasonal patterns in the amount of visitors over the course of the year (see Figure 1). Plausible values for the parameters are $p = 400$, $C = 400$, and $k = 200$. After the parameters p, k, and C are determined by the students, annual graphs of the revenue as a function of time can be generated.[6]

A periodic model for visitors, $v(t)$, across the course of year is the following

$$v(t) = 10 - 10\cos\left(\frac{2\pi}{365}t\right) \tag{3}$$

Utilizing Equation 3 and the given values for p, k, and C, and Equation 3, Figure 1 displays the revenue shortfalls and gains during the course of the year.

Follow on questions could include determining optimum values of the revenue as a context for understanding the chain rule and related rates. This part could be concluded with a reflective assignment or group discussion to evaluate the feasibility of the revenue model for the cooperative in terms of other cooperative functions such as the coffee harvest.

Attracting Visitors to the Cooperative

This part of the module builds on the foundation of the revenue model to engage students to investigate the role of mathematics to

[5] The question of a living wage was one that was recommended to the cooperative that has many social justice implications. Once p and k are determined from the data, it is an interesting question to analyze what type of living wage is meaningful and still allows the cooperative to have $R > 0$

[6] An assignment having students creating, evaluating the plausibility, and analyzing different models (constant, quadratic, periodic, etc) provides a good opportunity to discuss modeling with functions.

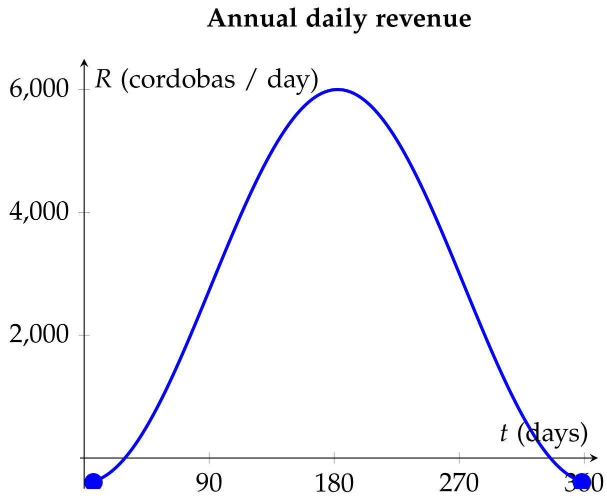

Figure 1: Dependence of the Cooperative's revenue throughout a year, given a conceptual model for the number of visitors. Equation 2, with $p = 670$, $k = 350$, $C = 400$, and $v(t) = -10\cos\left(\frac{2\pi t}{365}\right) + 10$.

model annual patterns in visitors to the cooperative. The cooperative has set a daily goal of 15-20 visitors to achieve a sustainable revenue stream. There are two modeling projects proposed for student investigation.

Under the assumption that marketing strategies through social media and other channels have been implemented and are increasing the number of visitors, students use the logistic model as a reasonable way to study the qualitative behavior of the rate of change of visitors to the cooperative over time. The logistic model in this context is formulated as:

$$\frac{dv}{dt} = r \cdot v \cdot (M - v) \tag{4}$$

where we have the following parameters:

$r =$ rate that knowledge about the cooperative spreads per day
$M =$ maximum number of people who know about the cooperative.

The objective of this analysis is for students to use the cooperative as a context to give meaning to the graphical relationship between a function and its derivative. Students should be able to use qualitative analysis of dv/dt to determine where $v(t)$ is increasing, increasing at an increasing rate, and increasing at a decreasing rate. It is important that students be asked to interpret their findings in a context and language useful to cooperative leaders. Students are asked to determine the equilibrium values of the model (that is, where $v(t)$ is not changing or when $dv/dt = 0$). Students are also asked to investigate the model in terms of the relationship between the rate of change of visitors and the increase in the number of visitors as a function of time.

Another extension is to use Google Trends, a data analytics site that allows students to explore publicly accessible aggregated search data from across the world, to investigate models for the rate of change of visitors to the cooperative. In a business setting, use of data and analytics is a driving force behind development of effective social media marketing campaigns.

To use Google Trends (http://www.google.com/trends/) students log in using a Google account. Students then type in a search term relevant to the cooperative that might be used in Google by people interested in an ecotourism experience like that offered by the cooperative. For each search term typed into Google Trends, a plot is generated by Google that shows the relative interest (scaled from 0 to 100) in the search term over the time period specified by the search.

For any search term, the data from the plot of relative interest over time can be downloaded as a comma separated (csv) file that can be opened in a spreadsheet application such as Excel or imported into a statistical software program such as R [7]. Students use the downloaded data to create a cumulative interest data set and then use the mathematical tools of the spreadsheet or other software to explore possible functions that could be used to model the data. Additional instructions that could be provided to the student are given in Appendix B.

[7] R Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, 2014. URL http://www.R-project.org/

Maximizing Revenue – Applied Optimization

The final project of the module builds on Parts I and II and develops the concept of customer growth rate. Revenue is revisited and redefined in terms of functions of v, the number of visitors per day to the cooperative.

More generally, we assume that there is a pool of potential visitors to the cooperative. This customer pool may fluctuate as interest in the cooperative increases or wanes (for example some visitors might not stay at the cooperative if it is too busy or overbooked).

A basic model that represents the growth rate of the customer pool can be given by Equation 4. It is convenient to use the notation $S(v)$ (with units of people/day) because the function $S(v)$ can be interpreted as a yield or supply function in an economic context [8]. A key mathematical assumption is that $S(v)$ is zero when $v = 0$ and some other positive nonzero value when $v \neq 0$.

[8] Colin W. Clark. *Mathematical Bioeconomics: The Mathematics of Conservation*. Wiley, 3rd edition, May 2010. ISBN 978-0-470-37299-9

Examination of the basic functional form of $S(v)$ as a quadratic function provides a context to understand optimization. Assume that the cooperative has a customer acquisition rate A (units visitors/day). This assumption modifies both the basic revenue model Equations 1 4 to the following:

$$\frac{dv}{dt} = S(v) - A \tag{5}$$
$$R(v) = p \cdot A - k(v) \cdot A - C, \tag{6}$$

where

$p =$	price charged per visitor (cordobas / visitor)
$A =$	supply of new visitors to the cooperative (visitors / day).
$k(v) =$	cost to accomodate vistors (cordobas / visitor).
$C =$	the fixed cooperative costs (cordobas / day) to accommodate visitors.

Note we generalize k to be a function of v from the linear model in Part II. In-class discussion could center on several different conceptual graphs of $S(v)$ with A, connecting the sign of dv/dt to growth/decline of the potential customer pool. This discussion is similar to the discussion of the basic revenue model in Equation 1.

Basic theory from economics suggests that in the absence of any other factors, dv/dt will be at equilibrium (dv/dt=0) when $S(v) = A$. Under this assumption, students can investigate the relationship of this economic model to applied optimization. First, the value of v that maximizes $S(v)$ is called the *maximum sustainable yield*. Second, the revenue model becomes:

$$R(v) = (p - k(v)) \cdot S(v) - C. \tag{7}$$

The value of v that optimizes $R(v)$ is called the *maximum economic yield*. Sample questions could include the following:

1. What relationship between p and $k(v)$ will be profitable? Why?
2. Consider the following customer growth function:

   $$S(v) = 0.05 \cdot v \cdot (30 - v). \tag{8}$$

   Determine the value of v that maximizes $S(v)$.
3. More generally, consider the function

   $$S(v) = r \cdot v \cdot (M - v), \tag{9}$$

   with r and M are constants. For this equation, determine the value of v that maximizes $S(v)$. Your final result should be an expression involving r and M. Check your work using the values of $r = 0.05$ and $K = 30$. Does the value of v that maximizes $S(v)$ match what you found in the previous problem?

4. Consider the revenue equation $R(v) = 670v - 300v - 300$. What is the value of v that maximizes R over the interval $0 \leq v \leq 30$? Explain this in context.

5. An equation for the per visitor cost is $k(v) = 100 \cdot 0.85^v + 200$. Sketch this function over the interval $0 \leq v \leq 30$. Give a contextual explanation for the behavior of this function, as compared to a fixed constant cost function of $k(v) = 300$.

6. Graph the general revenue equation $R(v) = (670 - 100 \cdot 0.85^v - 200)(0.05v \cdot (30 - v)) - 300$ with computational software (a plot of this function is shown in Figure 2).

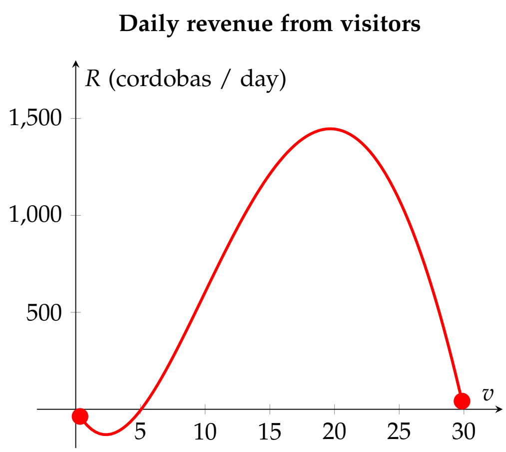

Figure 2: Dependence of the Cooperative's revenue on the number of daily visitors, using variable per visitor costs

What is the value of v that maximizes the revenue function $R(v)$? How does this compare to the value of v (the *maximum sustainable yield*) that maximizes $S(v)$?

7. Theory from mathematical bioeconomics suggests that if the maximum economic yield is greater than the maximum sustainable yield, then the resource (in this case visitors to the cooperative) is over-extended, leading to a ultimate decline in visitors. What are the implications for this result in terms of the long-term sustainability of the cooperative?

This part of the module could be concluded with a reflective assignment or group discussion on the differences between the the maximum sustainable yield and the maximum economic yield. Other optional extensions are simulation of the coupled system of equations with student developed models for customer acquisition A and effect on the annual revenue for the cooperative.

Several of the examples and graphs shown in this module could also apply when studying application and contexts for integration (e.g., Figures 1 and 2).

Analyzing the carbon footprint of the cooperative with remote sensing products

Remote sensing is the study of collecting information about the earth from sensors on aircraft or satellite. NASA has two long-term satellites (launched in 1999 and 2002, collectively called MODIS) with sensors that capture reflectance data at different wavelengths (termed "bands") and transmit it back to earth. The data in each wavelength or band is then processed to a specific data product for scientific use. Chances are you may have already seen images from MODIS - try googling "MODIS image of the day" for some examples.)

This module is appropriate to tie into investigations of functions, data fitting, and properties of the derivative. The remote sensing data accessed are provided on an eight-day timescale, which tends to be very messy at first. The structure of this project emphasizes data discovery and analysis with subsequent refinement of results using averages across months from the original data on an eight-day timescale.

The data are freely available and downloaded from NASA[9], and can be referenced to several locations across the globe. The web interface also provides easy visualization of the data as well as code to generate figures. The software package R[10] was used to process data, and instructions to access the remote sensing data are provided in (see Appendix D).

[9] ORNL Distributed Active Archive Center. Modis collection 5 global subsetting and visualization tool. 2014. DOI: 10.3334/ORNLDAAC/1241. URL http://dx.doi.org/10.3334/ORNLDAAC/1241

[10] R Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, 2014. URL http://www.R-project.org/

Five different remote sensing products are examined in this module:

- **Leaf Area Index (*LAI*)**: this remote sensing product is measured every 8 days at 1-kilometer resolution on a sinusoidal grid across the globe. The LAI variable defines the number of equivalent layers of leaves relative to a unit of ground area, so a *LAI* measurement of 2 means for every square meter of ground, there are 2 square meters of leaves. The units of LAI are measured as a ratio:

$$\frac{m^2 \text{ of leaf area}}{m^2 \text{ ground area}}$$

The LAI product is used to calculate land energy, information about the carbon and water cycle, and other vegetation characteristics.

- **Land surface temperature (LST)**: The remote sensing product land surface temperature (LST) measures how hot land is to the touch. It is calculated from measured emissivity of the land surface using concepts from physics. Land surface temperature is a measurement of how hot the land is to the touch. The data product is measured every 8 days at a 1 kilometer resolution on a sinusoidal grid across the globe. The units of land surface temperature are Kelvins, which are then converted to degrees Celsius. The measurement of land surface temperature is used for other measures of land-cover, water use, and other vegetation characteristics.
- **Gross Primary Productivity (GPP):** is a measurement of atmospheric carbon taken up by plants through the process of photosynthesis. As you recall from biology, plants use carbon dioxide and sunlight to produce simple sugar. (The other byproduct, oxygen, is one that is essential for humans.) Units of GPP are kilograms of carbon per m^2 per eight days.
- **Net photosynthesis (PSN):** is the difference between GPP and carbon lost through respiration. This is called *maintenance respiration* because it is carbon dioxide produced by maintaining normal structural processes in the plant. Units of PSN are kilograms of carbon per m^2 per eight days.
- **Evapotranspiration (ET)** is water lost to the atmosphere through evaporation from the soil and rain water that does not reach the ground (it is intercepted by the plant canopy). Units of ET are millimeters of H_2O per m^2 per eight days.
- **Potential evapotranspiration (PET)** is the maximal amount of evaporation that could occur for the given environmental conditions (temperature and humidity play a role in determining the total amount). Units of PET are millimeters of H_2O per m^2 per eight days.

To facilitate student analysis, all densities are scaled to the area of a farm on the coffee cooperative, calculated as 3.7 hectares (37000 m^2).

Analyzing the remote sensing products

This first part uses the land surface temperature and leaf area index data exclusively. We have found that students have an intuitive understanding of temperature and notion of leaf area index. Figure 3 shows the time series of land surface temperature for a particular cooperative farm over the course of 14 years.

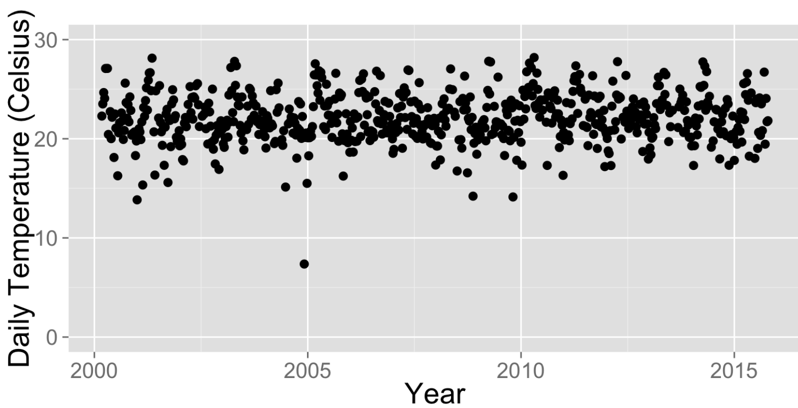

Figure 3: Remote sensing measurements of daily temperature for a coffee plantation in Peñas Blancas, Nicaragua

Students are asked to address the following questions about the data:

- What is the range of observed land surface temperatures?
- Do the remote sensing products contain any interesting behavior or trends (maximum/minimum values, periodic behavior, any other interesting facets to the data)?
- Choose just one years worth of data. Does limiting the window for plotting reveal any interesting behavior or trends?

These initial investigations usually display data that is “messy” - meaning that it doesn’t necessarily follow a nice smooth pattern from the functions that students may be accustomed to. Using either Microsoft Excel or any other software we ask students to fit different function families (linear, quadratic, cubic, exponential, logarithmic) to the year’s worth of data. While this isn’t a statistics course, some qualitative discussion of metrics to discuss goodness of fit (such as R^2 values) is useful. The goal is not to fit an exact model, but rather to have students visually make the connection between data and smooth functions.

A second approach is to examine all of the data from the corresponding months (across all years) and average the measurements. This provides a representative annual sample pattern of a measurement, as shown in Figure 4. Students can then also fit functions to the annual average pattern.[11]

[11] A project for a statistics class could be to examine the distribution of measurements (mean, median, 95% confidence interval) to get a sense of the monthly variability in each measurement.

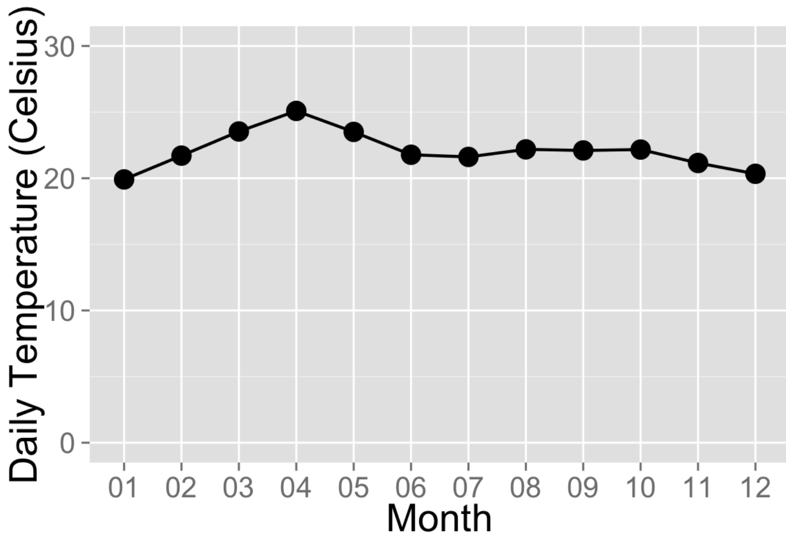

Figure 4: Annual monthly average of daily temperature for a coffee plantation in Peñas Blancas, Nicaragua

From the fitted function, guiding questions we ask students to report and reflect on include:

- If one was to try to fit a curve through the data for a best possible match, what function family would you choose and why?
- What are the time intervals where the fitted function is increasing/decreasing? Are there any times where the function is a maximum?
- Go back to the time intervals you identified above. What would they mean in the context of the specific data product? Where is the data product increasing the fastest? Decreasing the fastest? Discuss with your partner and record your thoughts below.

Working with remote sensing products that are rates

A second part of this project moves beyond the measurements of leaf area and land surface temperature to investigate the remote sensing products net photosynthesis and evapotranspiration, which are representative of how much carbon and water is being exchanged with the surface. The data are on the same timescale as land surface temperature and leaf area index, but are representative of the rate of change, rather than an absolute measurement. This provides a nice extension of the application of rates of change and interpretation of the derivative. As with the first part, it is helpful to show both the raw data, and the representative pattern of a measurement across a year, first by integrating all the rate measurements in a given month (so we find the total monthly amount of a measurement), and then averaging across corresponding years (to get a representative sample). A sample average pattern of the water released by this ecosystem through evapotranspiration (the remote sensing product ET) and the potential amount of water that can be lost (PET) is shown in Figure 5.

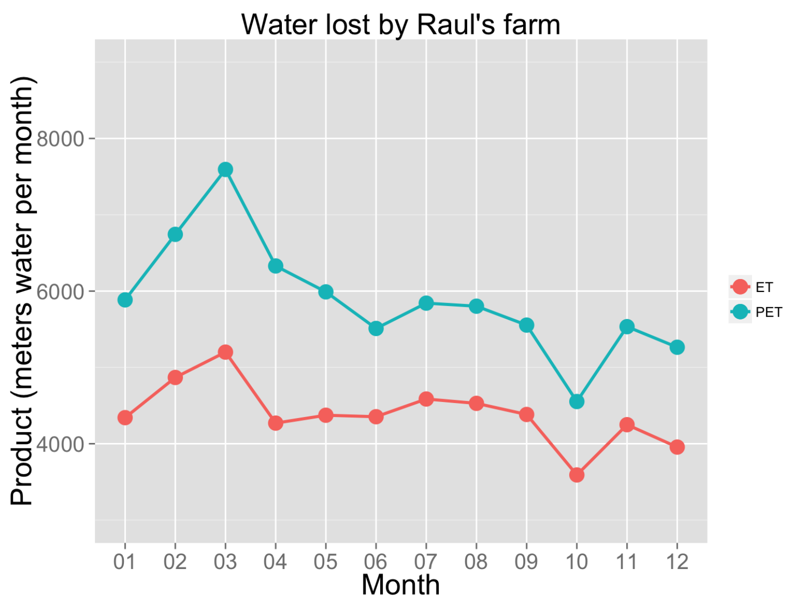

Figure 5: Annual pattern of evapotranspiration (ET, total water lost) and potential evapotranspiration (PET) for a farm in Peñas Blancas, Nicaragua

Questions we ask the students include:

- Make a plot of all of the data products over the time intervals. Are there any interesting trends or patterns in the data? *Note: students may have to zero in on a year or two first, rather than just looking at the entire time period.*
- Examine the data to determine when the coffee harvest occurs. Explain how this is possible from visual inspection of the remote sensing product(s).
- Construct a function that calculates the amount of carbon lost through maintenance respiration from knowing the data products GPP and PSN. **We will call the new data created with this function the data product RM.**
- With the remote sensing products ET and PET, form the ratio of ET/PET and describe how it can be related to the water stress of the ecosystem.[12]
- For each of the remote sensing products (GPP, PSN, RM, ET, PET, ET/PET), describe time intervals when they are increasing, decreasing, as well as critical and inflection points.

[12] If the ratio ET/PET is closer to zero, that suggests the ecosystem is undergoing drought stress, see http://www.ntsg.umt.edu/project/dsi

An annual pattern of GPP and PSN is shown in Figure 6. Note that net photosynthesis (PSN) will always be less than primary productivity (GPP) because it also accounts for any carbon loss during structural maintenance of the plants.

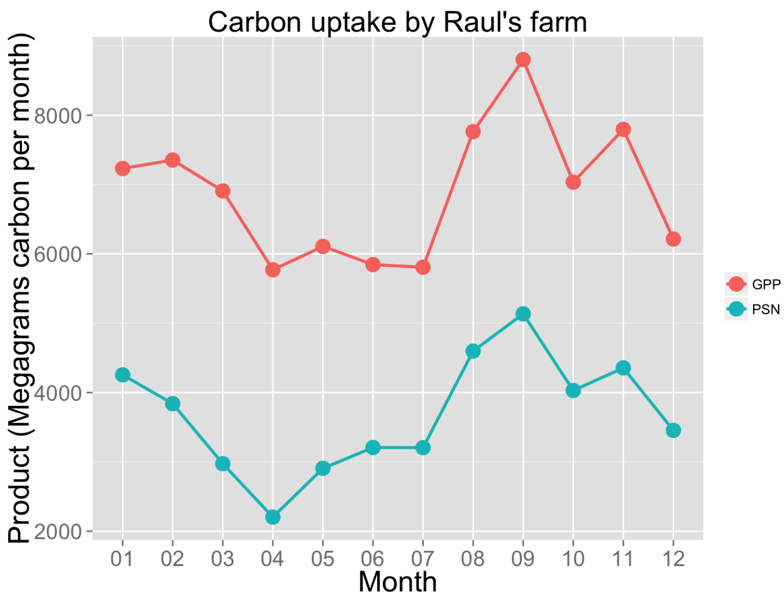

Figure 6: Annual pattern of primary productivity (GPP, total carbon absorbed) and net photosynthesis (PSN, carbon absorbed less respiration costs) for a farm in Peñas Blancas, Nicaragua

At this point a reflective assignment could be incorporated where students give a succinct and non-technical explanation of each remote sensing product for a cooperative member. A key question that we have students analyze with these data is the following:

How would you characterize this farm's annual carbon footprint from agricultural processes? Evaluate this question in relation to the carbon uptake and water usage from the remote sensing products.

Students responded to the above question as a cumulative assignment. What follows below is a synthesis of student responses edited to be an executive summary for the cooperative:

> Our results demonstrate that when carbon and water dynamics change, they are changing at a rapid rate. The average maximum amount of carbon attained by plants was 8800 Megagrams carbon in September, and the average minimum amount of carbon attained by plants was 5770 Megagrams in April. The behavior of the remote sensing products analyzed were also inconsistent with carbon values increasing and decreasing because of plants being grown and harvested. As the plants are growing, more carbon is being taken up, typically November to February.
>
> We would estimate that annual carbon footprint follows a pattern for an agricultural crop. This can be seen by the graphs intervals where

the function is increasing which means there is more carbon uptake since the harvest has not yet taken place. We can also see that when there is a dip in the carbon uptake it follows the harvest months of November to February.

A proper agricultural process would show signs of stable or increasing trends of water being evaporated, because more water would then be recycled. If the ratio of ET to PET is zero then that means that the cooperatives are not using the water efficiently. A drought weakens the land's ability to take in carbon.

Overall, the cooperatives' agricultural processes are generally stable. From the results described above, it appears when all the crops are planted at the start of harvest, the more water is being absorbed and the less water is being evaporated. Likewise, the less crops there are, such as at the end of harvest, there is less water absorbed by plants and more water is being evaporated to the atmosphere. If the primary source of water comes solely from rainfall then the cooperative is minimizing their carbon emissions by not using energy to bring water to their crops. If the cooperative does use energy to bring water to their crops, then the cooperative is losing a significant amount of water year-round, but especially right after harvest.

In conclusion, students were able to connect the remote sensing measurements to agricultural processes and identify an expected pattern at a farm in the GARBO cooperative. Future work to estimate the carbon footprint of the cooperative could expand this analysis to other farms located on the cooperative, or through additional remote sensing products.

Appendix A: Links to internet resources and data from the cooperative

Internet resources on the Peñas Blancas region:

- http://www.lonelyplanet.com/nicaragua/la-dalia-penas-blancas
- http://www.nicaragua.com/blog/visiting-the-spectacular-penas-blancas-massif
- https://vianica.com/go/specials/26-penas-blancas-massif-nicaragua.html

YouTube videos of the cooperative:

- https://www.youtube.com/watch?v=rOr8bhIqgzs
- https://www.youtube.com/watch?v=j4pkwNpxY2I
- https://www.youtube.com/watch?v=sLRbrL_0TRg
- https://www.youtube.com/watch?v=pXUklmPVIrE

Internet resources on remote sensing:

- https://en.wikipedia.org/wiki/Moderate-Resolution_Imaging_Spectroradiometer
- http://oceanservice.noaa.gov/facts/remotesensing.html
- http://earthobservatory.nasa.gov/Features/RemoteSensing/
- https://lpdaac.usgs.gov/dataset_discovery/modis/modis_products_table/mod15a2
- https://en.wikipedia.org/wiki/Leaf_area_index
- http://earthobservatory.nasa.gov/GlobalMaps/view.php?d1=MOD13A2_M_NDVI
- http://earthobservatory.nasa.gov/GlobalMaps/view.php?d1=MOD11C1_M_LSTDA

Common costs of items used by the cooperative:
Here is a table of common costs of things needed to accommodate guests from a visit to the cooperative in March 2012, as provided by cooperative leaders:

Item	**Cost**	**Notes**
Food		
Rice	10 cordobas / pound	1 pound feeds six people
Beans	8 cordobas / pound	1 pound feeds six people
Oil	18 cordobas / 0.25 Liters	Used for cooking
Tortillas	2 cordobas / 1 pound corn	1 batch makes 10 tortillas
Chicken	120 cordobas / hen	
Eggs	3 cordobas / egg	
Mango	10 cordobas	
Watermelon	100 cordobas	
Cantaloupe	40 cordobas	
Bread	29 cordobas	Price for 1 loaf
Pasta	8-10 cordobas	1 package feeds six people
Sugar	9 cordobas	
Cheese	35 cordobas	
Coffee	Cost of labor	
Purified water	20 cordobas / 500 mL	Needed for food cooking and consumption
Consumables		
Toilet paper	12 cordobas / roll	
Napkins	35 cordobas / package	A package has 60
Candles	2 cordobas / candle	1 candle lasts approximately 1.5 hours
Body soap	14 cordobas / bar	
Laundry soap	16 cordobas / load	
Matches	1 cordoba / box	The cooperative does not have a reliable source of electricity.
Batteries	25 cordobas / battery	

A website for market prices of common items in Nicaragua is:

http://www.numbeo.com/cost-of-living/country_result.jsp?country=Nicaragua

If you navigate to this website, the nearest city to the cooperative is Matagalpa.

Appendix B: Accessing Google trends

Google trends is a website that allows you to analyze data used in Google search engines. In order to use this you will need to log in with a Google account.

1. Navigate to Google Trends http://www.google.com/trends/
2. Type a search term in the topics menu bar. A plot should be generated highlighting the relative interest (scaled between 0 to 100) in this search term over time. The data are scaled in this way so searches in countries with more active internet users would not dominate the results. If you are curious for more information about how the data are scaled, please visit the following link: https://support.google.com/trends/answer/4365533?hl=en&rd=1
3. Select a search term that will be of relevant for the cooperative in terms of their marketing (e.g. knowing the trends in how Google users search for "Game of Thrones" isn't useful to the cooperative but "winter escape" might be, especially for users in colder winter climates.)
4. For ease of use in visualization, limit the results to one year. To do that, click on "select time range" and select a particular year.
5. To download the data, at the top right of the webpage you should see a gear icon, allowing you to download the data as as file "Download as CSV". When you do this, you will download a **c**omma **s**eparated **v**alues file called "report.csv". Save this csv file and open it. When you open up this file in a spreadsheet program, the first few lines provide information about the web search. Starting at about line 6 is the "interest" data graphed on the webpage. Notice that in column "A" there is a time stamp corresponding to the weeks. It maybe be helpful to re-write this as weeks of the year.
   (a) First, create a column labeled Weeks as follows:
   (b) In cell A6 type the following formula in quotations: "=1"
   (c) In cell A7 type the following: "=A6+1".
   (d) Now, click and drag the lower right corner of cell A7 down the

length of the column. This will autofill the formula down. The last cell, A57, should read 52.

Appendix C: Accessing remote sensing products

Remote sensing products can be downloaded via a NASA web server co-located to a particular latitude and longitude [13]. The web service provides basic visualization of the remote sensing products as well as scripts for use in R [14] and RStudio [15]. The following instructions were accurate as of December 2015:

[13] ORNL Distributed Active Archive Center. Modis collection 5 global subsetting and visualization tool. 2014. DOI: 10.3334/ORNLDAAC/1241. URL http://dx.doi.org/10.3334/ORNLDAAC/1241

[14] R Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, 2014. URL http://www.R-project.org/

[15] RStudio Team. *RStudio: Integrated Development Environment for R*. RStudio, Inc., Boston, MA, 2015. URL http://www.rstudio.com/

1. Navigate to Global Subsetting Tool: MODIS Land Products http://daacmodis.ornl.gov/cgi-bin/MODIS/GLBVIZ_1_Glb/modis_subset_order_global_col5.pl

2. Select a latitude or longitude of the location (or the center pixel) where you wish to download data. The GARBO coffee cooperative is at Latitude [13.2708] Longitude [-85.71935]

3. Select a remote sensing product that you would like to analyze. Notice the specific code associate with each remote sensing product (MOD16A2 = Evapotranspiration) More information about the MODIS data products can be found at https://lpdaac.usgs.gov/dataset_discovery/modis, and a table of all the remote sensing products with additional information is located at https://lpdaac.usgs.gov/dataset_discovery/modis/modis_products_table

4. Once you have selected your remote sensing product and spatial coverage, you can select a time window for the data products.

5. Following a confirmation page, you will be sent an email when the data are available for download, provided as a link. After data are ready, you will be taken to a page where you can access basic plots of the data or download R scripts.

Appendix D: Notes for module delivery

When presenting a module, tasks were structured both as individual homework assignments as well as pairs of students working together to complete the mathematics and present written summaries of results. The greatest success in use of this model has come from implementing the module in smaller pieces (three parts over the course of the term) rather than one large project together. This helps distribute the grading and provides an opportunity for students to develop their skills in writing and preparation of the reports. The module, however, could definitely serve as an independent study project over the course of the term.

For grading and evaluation of student work, we recommend a developmental approach as students complete project tasks. Because there are other aspects to a calculus course (homework, exams, etc), we have found it effective to have students submit work in pairs along with any pencil and paper work needed in their calculations. Shorter write ups (no more than two pages) per each individual part work best in terms of student willingness to complete as well as efficiency in instructor grading. Each part of the module is graded on a simple rubric developed by the instructor and the expectations of writing and point values increase over the duration of the project (10 points for the first part, 20 for the second, and 30 for the third). In addition to the grading rubric, qualitative comments were provided to motivate improved performance as student groups progressed to the next tasks of the project.

Bibliography

ORNL Distributed Active Archive Center. Modis collection 5 global subsetting and visualization tool. 2014. DOI: 10.3334/ORNL-DAAC/1241. URL http://dx.doi.org/10.3334/ORNLDAAC/1241.

Colin W. Clark. *Mathematical Bioeconomics: The Mathematics of Conservation*. Wiley, 3rd edition, May 2010. ISBN 978-0-470-37299-9.

Peter Levine and Karol Edward Soltan, editors. *Civic Studies*. The Civic Series. January 2014.

R Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, 2014. URL http://www.R-project.org/.

RStudio Team. *RStudio: Integrated Development Environment for R*. RStudio, Inc., Boston, MA, 2015. URL http://www.rstudio.com/.

Carol Schumacher, Martha Siegel, and Paul Zorn. *2015 CUPM Curriculum Guide to Majors in the Mathematical Sciences*. Mathematical Association of America, 2015. ISBN 978-0-88385-917-9.