1. Modeling Exercises

1) [This problem involves data averaging/smoothing.]

You are given the following table of raw temperature data.

Year	°C	5-year Average	Gaussian
1988	20		
1989	21		
1990	19		
1991	20.5		
1992	20		
1993	18.5		
1994	21		
1995	20		
1996	19.5		
1997	20.5		
1998	21.25		

(a) Plot the temperature data and connect with lines and then sketch a smooth curve that seems to approximate the data pretty well; also, predict the value of the temperature in the year 2000.

(b) Use the data table and compute the 5-year rolling average of the data for 1990-96. Show one such calculation. Plot your results and use them to predict the value of the temperature in the year 2000.

(c) Use the temperature data and the weights below to compute Gaussian averages for 1990-96. Show one such calculation. Plot your results and use your results to predict the temperature for the year 2000. Use the weights (0.06, 0.24, 0.4, 0.24, 0.06).

(d) Why might one want to report data that have been averaged as in (b) and (c) rather than plotting raw data?

2) [Refer to the chapter by Copp and Zanella on "The Greenhouse Effect: Revolution Involves Risk."]

(a) Based upon the chemical equation for the combustion of octane given in this chapter, estimate the number of tons of CO_2 produced each year by automobiles in the United States. (Hint: There are 2.65 kilograms of octane per gallon, and 1000 kilograms per ton.)
(b) How many trees would have to be planted to offset the CO_2 produced by cars in the US? Assume each new tree uses 10 kilograms of CO_2 per year.
(c) Another strategy for reducing CO_2 output from transportation is to increase the fuel economy for automobiles. How much CO_2 would be "saved" each year by requiring the nation's cars to average 50 miles per gallon?
(d) Of the two approaches for lowering CO_2 described above, which one would you choose? Clearly state both the quantitative reasons and the value judgments for your decision.

3) [This question is about modeling the Earth's energy balance.]

(a) Data suggest that the average surface temperature of the Earth has increased since 1860 by about 0.5 °C. However, one of the suggested articles mentions that the increase is about 1 °F. Is there a contradiction here? Explain.

(b) Energy balance between the amount received by the Earth from the Sun and the amount radiated by the Earth (using the reasonable assumption that the Earth behaves as a blackbody) results in the following expression:

$$T = \left(\frac{S}{4\cdot\sigma}\right)^{1/4}, \qquad (1)$$

where T is the Earth's temperature in Kelvin, S is the solar "constant" (about 1360 W/m^2, i.e., 1360 Watts per squared meter), and σ is a fundamental constant of physics ($5.67\cdot10^{-8}$ W/m$^2\cdot$K^4). Derive equation (1), carefully explaining all steps and assumptions.

(c) Using equation (1), show that the temperature of the Earth's surface should be about 5 °C, instead of the more accurate 15 °C. This difference is the result of assumptions we made in deriving equation (1). If we take into account that some of the solar radiation is reflected back in space, mostly by the clouds, we obtain

$$T = \left(\frac{0.7\cdot S}{4\cdot\sigma}\right)^{1/4}, \qquad (2)$$

while if we also take into account that clouds reradiated back to the Earth a large percentage of the Earth's infrared radiation, we obtain

$$T = \left(\frac{1.15\cdot S}{4\cdot\sigma}\right)^{1/4}. \qquad (3)$$

Show <u>how to set up</u> the energy balance equation to derive equations (2) and (3). Using equation (2), show that the Earth's temperature is about −19 °C (which agrees very well with the measured value of the Earth's spacebound radiation from high-altitude satellites). Using the more accurate equation (3), show that the Earth's surface temperature is about 15 °C, in good agreement with observations.

(d) Use the following data, plot a graph and use it to estimate what the average global temperature might be in the year 2040. Assume that a doubling of atmospheric CO_2 compared to the pre-industrial level will cause the temperature to rise by 3°C. What other assumptions have you used in extrapolating your graph? Why is it likely that your estimate is low?

Year	**CO_2 (ppm)**
Pre-industrial	280
1860	270–290
1958	310–315
1970	325
1975	330
1982	339
1988	350

(e) Comparing your result from (d) to equation (3), find the increase in the percentage of energy reradiated back to the Earth due to CO_2.

(f) How does your answer in (e) compare to the natural variation in S of about 0.1–0.2%, which last a couple of weeks? Should we worry more about CO_2 or about what is happening in the Sun? What if the variations in S lasted for years?

4) [This question is about the rate of crater formations due to collisions.]

In class we determined from lunar crater counting that about 80,000 years on average passes between impacts on Earth with objects that are 0.1–0.5 km in diameter. It is also hypothesized (see the article "Target Earth: It *Will* Happen"; p25 of the handout) that the number of craters of a given size varies approximately as the inverse of the projectile's diameter squared. Put another way, this means that the time between impacts is proportional to the square of the diameter of the projectile. This question relates to finding the frequency/risk of smaller projectiles in the range of 0.01–0.05 km (10–50 m) diameter from the above information/hypothesis. Note that these projectiles still have energies in the range of 1–10 hydrogen bombs. Ignore the effect of the Earth's atmosphere on these objects.

(a) Estimate the number of years between impacts from objects in the 10-50 m diameter size over the whole Earth.

(b) Estimate the number of years between impacts from such objects over
  (i) the continental United States;
  (ii) the state of Wisconsin;
  (iii) Marathon County.

(c) Let's say that this type of impact would cause 200 billion dollars worth of property damage (very rough estimate).
  (i) Using the numbers from (2.b.ii), estimate how many dollars per year this works out to be for the state of Wisconsin if the damage cost is averaged out over the time between impacts.
  (ii) Given that there are about 5 million people living in Wisconsin, how many dollars per year per person would this amount to for the state? Your answer is a ball park estimate of the insurance premium that might be paid to cover this type of catastrophe.

5) [This question is about population growth and modeling in science.]

Pick a country of your choice from the 1997 World Population Data Sheet. Find the 1997 population and the natural % increase rate in column four.

(a) Use the exponential growth model to predict populations for the years 1997–2025.
(b) Follow the logistic growth model example in the class handout to predict the population for 1997–2025 assuming logistic growth. Use a reasonable value of twice the current population for the carrying capacity of the country you have chosen. Note that when you do this the value r in the logistic model should be twice the value used above from column 4.
(c) Plot both models' predicted populations on the same graph.
(d) Answer the following questions (related in general to human populations) in a few short paragraphs.
  i. Explain why exponential growth cannot go on forever.
  ii. Give two examples of mechanisms by which large populations can cause population growth rates to decline.
  iii. For each doubling of population, what services needed by a society would also need to double in order for a country to maintain its standard of living? List five such services.

6) [This question is about the Alvarez model.]

The physicist Luis Alvarez, and his geologist son Walter, proposed a radical and controversial theory to explain the disappearance of dinosaurs 65 million years ago.

(a) On which main observation was their proposal based?

(b) What were the four main predictions of their hypothesis?

(c) Did the tests of these predictions support their theory?

(d) In order to simplify our calculations, let us assume that for a long time after a collision between an asteroid and a planet, we find a 5-cm wide layer of leftover debris around the planet. The radius of the planet is R = 6,400 km. If the density in the layer is about 5 times that of water, how much mass is in the layer?

(e) Most of the material in the layer comes from the surface of the planet. After the blast, this material was thrown up in the atmosphere and after a period of time was deposited throughout the surface of the planet. It is reasonable to assume that only about 1% of the material in the layer was the mass of the asteroid. Assuming again that the average density of the asteroid was 5 times that of water, find the asteroid's radius. Observations suggest that the radius of the resulting crater is about 20 times the radius of the asteroid. How big is the resulting crater in this case?

(f) If this asteroid's speed at collision was a very low 15 km/s, how much energy was released during the collision? How many Hiroshima bombs is this energy equivalent to?

7) [This question is about the different steps in a collision between Earth and an asteroid.]

(a) You are using an optical telescope to find potentially threatening, for our planet, asteroids. The resolution of your telescope is 0.1 arc seconds. What is the closest distance at which you could clearly see an asteroid of diameter 1 km? In reality, we could spot this asteroid at a much farther distance. Explain how this is possible.

(b) Imagine that you spotted this asteroid at a distance of 20 million kilometers. Initial observations suggest that this object is in a direct collision course with Earth. How much warning time do we have before collision if the average speed of the asteroid is about 15 km/s?

(c) The exact path of an asteroid is very uncertain unless one has observations spanning many months. After only a few day's worth of observations, astronomers suggest that even though it is very likely that a collision will occur, the uncertainty in the exact path of the asteroid is about 0.5°. Is it possible that this asteroid will collide with the Moon instead of the Earth? Show your calculations! (The distance between Earth and Moon is about 400,000 km. Assume that the Moon's orbit around the Earth is on the same plane as that of the Earth's orbit around the Sun. In reality the latter is not true but we make this assumption for simplicity.)

(d) Assume that this is a stony-iron asteroid with average density five times that of water. Ignore the effect of our atmosphere on the asteroid, and assume that the collision speed is about 40 km/s. (The range of collision speeds is 10–70 km/s, depending on where the Earth is located on its orbit at the time of the collision.) What is the energy (in Joules) delivered upon collision? What is the equivalent in megatons?

(e) Assume that the impact site is somewhere in the Atlantic. The energy needed to evaporate one kilogram of water (from 0° C water to 100° C steam) is about $2.7 \cdot 10^6$ Joules. What is the size of the cubic volume of ocean water that will evaporate? (That is, if I were to collect all that water that will evaporate in a cubic container, you need to find the size of the cube.)